OCTOBER 2016

$Time - Three\ hours$

$(Maximum\ Marks: 75)$

[N.B:- (1) Answer any FIVE questions in each of PART-A& PART-B and any two divisions of each questions in PART-C

(2) Each questions carries 2(two)marks in PART-A ,3(three)marks in PART-B and 5(five)marks for each division in PART-C.]

PART – A

1. Define discrete random variable
2. If $E(x) = 3$, find $E\,(2\,x + 5)$
3. If $n = 10$, $p = \frac{1}{2}$,find the mean and variance of a binomial distribution
4. Give two properties of normal distribution
5. Write the condition for minimum of the function $y = f(x)$ at $x = a$
6. Find the area bounded by the curve $y = \frac{1}{x}$, the x-axis and the ordinates $x = 1$ and $x = 2$
7. Write down the integrating factor of $\frac{dy}{dx} + y\cot x = \operatorname{cosec} x$
8. Write down the auxiliary equation of $4\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 9y = 0$

PART – B

9. If a random variable 'X' has the following probability distribution , find E(X)

X	-3	6	9
$P(X)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

10. If the variance of a Poisson distribution is 0.35 ,find $P(X = 3)$

11. Write the normal equations to fit a straight line $y = mx + c$

12. The distance travelled by a particle in given by $s = 2t^3 - 3t^2 + 1$ in 't' secs . Find the initial velocity and initial acceleration of the particle

13. Find the slope of the tangent to the curve $y^2 = 4ax$ at $(at^2, 2at)$

14. Solve: $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

15. Find the Particular Integral of $(D^2 - 3D + 2)y = e^{-x}$

16. Find the complementary function of $(D^2 - 100)y = \cos 9x$

PART- C

17.(a) A random variable 'X' has the following probability distribution

X	0	1	2	3	4
P (X)	3a	4a	6a	7a	8a

Find (i) The value of 'a' and (ii) $P(X \geq 3)$

(b) A random variable 'X' has the following probability distribution.

X	0	1	2	3
$P(X)$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$

Find mean and variance

(c) The mean and variance of a binomial distribution are 16 and 8, Find $P(X = 0)$ and $P(X = 1)$

18.(a) If $'X'$ is a poission variate such that $P(X = 1) = 0.3$ and $P(X = 2) = 0.1$ find $P(X = 0)$

(b) If $'X'$ is normally distributed with mean 80 and standard deviation 10 , find $P(70 \le X \le 100)$

Take $P(0 \le z \le 1) = 0.3413, P(0 \le z \le 2) = 0.4772$

(c) Fit a straight line y=ax+b for the following data

x	0	1	2	3	4
y	1	1	3	4	6

19.(a) Find the velocity when the acceleration is zero, if the distance travelled by a particle 's' is given by

$s = t^3 - 6t^2 + 12t - 8$

(b) Find the equation of the normal to the curve $y^2 = 4x$ at (4 , 4)

(c) Find the maximum and minimum of $y = 2x^3 + 3x^2 - 36x + 1$

20. (a) Find the volume of a right circular cone of height '□' and radius $'r'$ by integration

(b) Solve: $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

(c) Solve: $\frac{dy}{dx} + \frac{y}{x} = x^2$

21. (a) Solve: $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

(b) Solve: $(D^2 + 3D + 2)y = 5e^{-4x}$

(c) Solve: $(D^2 + 9)y = \cos 2x$

ANSWERS

PART - A

1. Define discrete random variable

Answer If a random variable 'X' takes only a countable number of values then X is said to be discrete random variable

2. If $E(x) = 3$, find $E(2x + 5)$

Answer

$$E(2x + 5) = E(2x) + E(5)$$
$$= 2E(x) + 5 \quad [\because E(c) = c]$$
$$= 2(3) + 5$$
$$= 6 + 5$$
$$= 11$$

3. In $n = 10$, $p = \frac{1}{2}$, find the mean and variance of a binomial distribution.

Answer Given $n = 10$, $p = \frac{1}{2}$

Since $q = 1 - p$

$$q = 1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$$

Mean $= np = 10\left(\frac{1}{2}\right) = 5$

Variance $= npq = 10\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{5}{2}$

4. Give two properties of normal distribution

Answer (i). The normal curve is bell shaped.

(ii). Mean = Median = Mode = μ

5. Write the condition for minimum of the function $y = f(x)$ at $x = a$

Answer

Condition for minimum

(i) $\frac{dy}{dx} = 0$ at $x = a$

(ii) $\frac{d^2y}{dx^2} = +ve \ (or) > 0$ at $x = a$

6. Find the area bounded by the curve $y = \frac{1}{x}$, the x-axis and the ordinates $x = 1$ and $x = 2$

Answer

$$Area = \int_a^b y\,dx$$

Here $y = \frac{1}{x}$, $a = 1, b = 2$

$$A = \int_1^2 \frac{1}{x}dx$$

$$= [\log x]_1^2$$

$$= \log 2 - \log 1$$

$$= \log 2 \ Sq.units$$

7. Write down the integrating factor of $\frac{dy}{dx} + y\cot x = \operatorname{cosec} x$

Answer

Given $\frac{dy}{dx} + y\cot x = \operatorname{cosec} x$

This is of the form $\frac{dy}{dx} + Py = Q$

Here, $P = \cot x = \frac{\cos x}{\sin x}$, $Q = \operatorname{cosec} x$

Integrating Factor $= e^{\int P dx}$

$$= e^{\int \frac{\cos x}{\sin x}dx}$$

$$= e^{\log(\sin x)} = \sin x$$

8. Write down the auxiliary equation of

$$4\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 9y = 0$$

Answer

Auxiliary equation is $4m^2 - 12m + 9 = 0$

PART - B

9. If a random variable 'X' has the following probability distribution , find E(X)

X	– 3	6	9
P (*X*)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Answer

Formula: $E(X) = \sum_{i=1}^{n} x_i p_i$

$$E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n$$

$$= \left(-3 \times \frac{1}{6}\right) + \left(6 \times \frac{1}{2}\right) + \left(9 \times \frac{1}{3}\right)$$

$$= \frac{-3}{6} + \frac{6}{2} + \frac{9}{3}$$

$$= \frac{-3(1) + 6(3) + 9(2)}{6}$$

$$= \frac{-3 + 18 + 18}{6}$$

$$= \frac{33}{6}$$

$$= \frac{11}{2}$$

10. If the variance of a poisson distribution is 0.35 ,find $P(X = 3)$

Answer

Formula: $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$,

Given $variance\ \lambda = 0.35$

$$P(X = x) = \frac{e^{-0.35}(0.35)^x}{x!}$$

$$P(X = 3) = \frac{e^{-0.35}(0.35)^3}{3!}$$

$$= \frac{e^{-0.35}(0.043)}{6}$$

11. Write the normal equations to fit a straight line

$y = mx + c$

Answer

Normal Equation are

$$m\sum x + nc = \sum y$$

$$m\sum x^2 + c\sum x = \sum xy$$

12. The distance travelled by a particle is given by

$s = 2t^3 - 3t^2 + 1$ in '*t*' secs . Find the initial velocity

and initial acceleration of the particle

Answer

$$s = 2t^3 - 3t^2 + 1$$

$$v = \frac{ds}{dt} = 2(3t^2) - 3(2t) + 0$$

$$= 6t^2 - 6t$$

$$a = \frac{d^2s}{dt^2} = 6(2t) - 6(1)$$

$$= 12t - 6$$

Initial velocity $v = \left(\frac{ds}{dt}\right)_{t=0} = 6(0)^2 - 6(0) = 0$ units / sec

Initial acceleration $a = \left(\frac{d^2s}{dt^2}\right)_{t=0} = 12(0) - 6 = -6$ units /sec^2

13. Find the slope of the tangent to the curve

$y^2 = 4ax$ at $(at^2, 2at)$

Answer

$$y^2 = 4ax$$

$$2y\,\frac{dy}{dx} = 4a(1)$$

$$\frac{dy}{dx} = \frac{4a}{2y}$$

$$\frac{dy}{dx} = \frac{2a}{y}$$

$$\left(\frac{dy}{dx}\right)_{(at^2,2at)} = \frac{2a}{2at} = \frac{1}{t}$$

$\therefore$ Slope of the tangent $m = \frac{1}{t}$

14. Solve: $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

Answer

Given $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\tan^{-1} y = \tan^{-1} x + C,$$

15. Find the particular integral of

$(D^2 - 3D + 2)y = e^{-x}$

Answer

Given $(D^2 - 3D + 2)y = e^{-x}$

$$P.I. = \frac{e^{-x}}{D^2-3D+2}$$

Replace D by -1

$$P.I. = \frac{e^{-x}}{(-1)^2-3(-1)+2}$$

$$P.I. = \frac{e^{-x}}{1+3+2}$$

$$= \frac{e^{-x}}{6}$$

16. Find the complementary function of
$$(D^2 - 100)y = \cos 9x$$

Answer

Given $(D^2 - 100)y = \cos 9x$

Auxiliary equation $m^2 - 100 = 0$

$$m^2 = 100$$

$$m = \pm\sqrt{100} = \pm 10$$

∴, Complementary Function $(C.F) = Ae^{10x} + Be^{-10x}$

PART - C

17.(a). A random variable 'X' has the following probability distribution

X	0	1	2	3	4
P (X)	3a	4a	6a	7a	8a

Find (i) The value of 'a' and (ii) $P(X \geq 3)$

Answer

(i) Formula: $\sum P_i = 1$

$$3a + 4a + 6a + 7a + 8a = 1$$

$$28a = 1$$

$$a = \frac{1}{28}$$

(ii) $P(X \geq 3) = P(X=3) + P(X=4)$

$= 7a + 8a$

$= 15a$

$= 15\left(\frac{1}{28}\right)$

$= \mathbf{\frac{15}{28}}$

17.(b). A random variable 'X' has the following probability distribution

X	0	1	2	3
$P(X)$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$

Find mean and variance

Answer

Formula: **Mean =** $E(X)$

Variance $= E(X^2) - [E(X)]^2$

$$E(X) = \sum_{i=1}^{n} x_i p_i$$

$$E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n$$

$$= \left(0 \times \frac{1}{3}\right) + \left(1 \times \frac{1}{6}\right) + \left(2 \times \frac{1}{6}\right) + \left(3 \times \frac{1}{3}\right)$$

$$= 0 + \frac{1}{6} + \frac{2}{6} + 1$$

$$= \frac{1+2+6}{6}$$

$$= \frac{9}{6}$$

Mean $= \frac{3}{2}$

$$E(X^2) = \sum_{i=1}^{n} x_i^2 \; P_i$$

$$E(X^2) = x_1^2p_1 + x_2^2p_2 + \cdots + x_n^2p_n$$

$$= \left(0^2 \times \frac{1}{3}\right) + \left(1^2 \times \frac{1}{6}\right) + \left(2^2 \times \frac{1}{6}\right) + \left(3^2 \times \frac{1}{3}\right)$$

$$= \left(\mathbf{0} \times \frac{1}{3}\right) + \left(\mathbf{1} \times \frac{1}{6}\right) + \left(\mathbf{4} \times \frac{1}{6}\right) + \left(\mathbf{9} \times \frac{1}{3}\right)$$

$$= 0 + \frac{1}{6} + \frac{4}{6} + 3$$

$$= \frac{1 + 4 + 18}{6}$$

$$= \frac{23}{6}$$

Variance $= \boldsymbol{E(X^2) - [E\,(X)]^2}$

$$= \frac{23}{6} - \left(\frac{3}{2}\right)^2$$

$$= \frac{23}{6} - \frac{9}{4}$$

$$= \frac{23(4) - 9(6)}{24}$$

$$= \frac{92 - 54}{24}$$

$$= \frac{\mathbf{38}}{\mathbf{24}}$$

$$= \frac{\mathbf{19}}{\mathbf{12}}$$

17.(c). The mean and variance of a binomial distribution are 16 and 8, Find $P(X = 0)$ and $P(X = 1)$

Answer

Given Mean: $np = 16 - - - - (1)$

Variance : $npq = 8 - - - - (2)$

$$\frac{(2)}{(1)} \Rightarrow \frac{npq}{np} = \frac{8}{16}$$

$$\boldsymbol{q = \frac{1}{2}}$$

We know that $p + q = 1$

$$p = 1 - q$$

$$= 1 - \frac{1}{2}$$

$$= \frac{2-1}{2}$$

$$\boldsymbol{p = \frac{1}{2}}$$

$$(1) \Longrightarrow \quad \text{np} = 16$$

$$n\left(\frac{1}{2}\right) = 16$$

$$\boldsymbol{n = 32}$$

Binomial distribution is $\boldsymbol{P(X = x) = nc_x p^x q^{n-x}}$

$$P(X = x) = 32c_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{32-x}$$

$$P(X = x) = 32c_x \left(\frac{1}{2}\right)^{x+32-x}$$

$$P(X = x) = 32c_x \left(\frac{1}{2}\right)^{32}$$

Put $x = 0$, $\quad P(X = 0) = 32c_0 \left(\frac{1}{2}\right)^{32}$

$$P(X = 0) = 1 \times \left(\frac{1}{2}\right)^{32}$$

$$P(X = 0) = \frac{1}{2^{32}}$$

Put $x = 1$, $\quad P(X = 1) = 32c_1 \left(\frac{1}{2}\right)^{32}$

$$= 32\left(\frac{1}{2^{32}}\right)$$

$$= 2^5\left(\frac{1}{2^{32}}\right)$$

$$= \frac{1}{2^{27}}$$

18.(a). If $'X'$ is a poission variate such that $P(X = 1) = 0.3$ and $P(X = 2) = 0.1$ find $P(X = 0)$

Answer Formula: $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given $\boldsymbol{P(X = 1) = 0.3}$

$$\frac{e^{-\lambda}\lambda^1}{1!} = 0.3$$

$$\frac{e^{-\lambda}\lambda}{1} = 0.3$$

$$e^{-\lambda}\lambda = 0.3 - - - - - (1)$$

Given $\boldsymbol{P(X = 2) = 0.1}$

$$\frac{e^{-\lambda}\lambda^2}{2!} = 0.1$$

$$\frac{e^{-\lambda}\lambda^2}{2} = 0.1$$

$$e^{-\lambda}\lambda^2 = 0.2 - - - - - (2)$$

$$\frac{(2)}{(1)} \Longrightarrow \frac{e^{-\lambda}\lambda^2}{e^{-\lambda}\lambda} = \frac{0.2}{0.3}$$

$$\lambda = \frac{0.2}{0.3}$$

$$\lambda = \frac{2}{3}$$

To find $\boldsymbol{P(X = 0)}$

$$P(X = 0) = \frac{e^{-\lambda}(\lambda)^0}{0!}$$

$$= \frac{e^{-\lambda}1}{1}$$

$$= e^{-\lambda}$$

$$= e^{-\frac{2}{3}} = 1.947$$

18.(b). If $'X'$ is normally distributed with mean 80 and standard deviation 10 , find $P\,(70 \le X \le 100)$

Take $P\,(0 \le z \le 1) = 0.3413, P\,(0 \le z \le 2) = 0.4772$

Answer Refer *October 2017 ,Question no:* 18(a) *,Page no:* 68

18.(c) Fit a straight line $y = ax + b$ for the following data

x	0	1	2	3	4
y	1	1	3	4	6

Answer

Let $y = ax + b$ ... (1) be the line of best fit.

Then the normal equations are

$$a\sum x_i + nb = \sum y_i \quad .... \quad (2)$$

$$a\sum {x_i}^2 + b\sum x_i = \sum x_i y_i \quad .... \quad (3)$$

We compute $\sum xi, \sum xi^2, \sum yi$ and $\sum xi\, yi$ from the following table.

xi	yi	xi^2	$xiyi$
0	1	0	0
1	1	1	1
2	3	4	6
3	4	9	12
4	6	16	24
$\sum x_i = 10$	$\sum y_i = 15$	$\sum x_i^2 = 30$	$\sum x_i y_i = 43$

Here, $n = 5$

Using the normal equations, we get

(2)⟹ $\quad \boldsymbol{a\sum x_i + nb = \sum y_i}$

$$a(10) + (5)b = 15$$

$$10a + 5b = 15$$

(3)$\Longrightarrow$ $a\sum x_i^2 + b\sum x_i = \sum x_i y_i$

$$a(30) + b(10) = 43$$

$$30a + 10b = 43$$

By Cramer's Rule : $10a + 5b = 15$ and $30a + 10b = 43$

$$\Delta = \begin{vmatrix} 10 & 5 \\ 30 & 10 \end{vmatrix} = 100 - 150 = -50$$

$$\Delta_a = \begin{vmatrix} 15 & 5 \\ 43 & 10 \end{vmatrix} = 150 - 215 = -65$$

$$\Delta_b = \begin{vmatrix} 10 & 15 \\ 30 & 43 \end{vmatrix} = 430 - 450 = -20$$

$$a = \frac{\Delta_a}{\Delta} = \frac{-65}{-50} = \mathbf{1.3}$$

$$b = \frac{\Delta_b}{\Delta} = \frac{-20}{-50} = \mathbf{0.4}$$

(1) $\Rightarrow y = ax + b$

Put $a = 1.3$ and $b = 0.4$

$\boldsymbol{y = 1.3x + 0.4}$, which is the line of best fit.

19.(a). Find the velocity when the acceleration is zero,

if the distance travelled by a particle 's' is given by

$s = t^3 - 6t^2 + 12t - 8$

Answer $s = t^3 - 6t^2 + 12t - 8$

$$v = \frac{ds}{dt} = 3t^2 - 6(2t) + 12(1) - 0 = 3t^2 - 12t + 12$$

$$a = \frac{d^2s}{dt^2} = 3(2t) - 12(1) + 0 = 6t - 12$$

To find the velocity when the acceleration is zero:

$$\Rightarrow a = 0$$

$$6t - 12 = 0$$

$$6t = 12$$

$$t = 2$$

when t = 2 secs ⇒ $v = 3(2)^2 - 12(2) + 12 = 0$ units / sec

19.(b). Find the equation of the normal to the curve

$y^2 = 4x$ at (4 , 4)

Answer

$$y^2 = 4x$$

$$2y\,\frac{dy}{dx} = 4(1)$$

$$\frac{dy}{dx} = \frac{4}{2y}$$

$$\frac{dy}{dx} = \frac{2}{y}$$

$$\left(\frac{dy}{dx}\right)_{(4,4)} = \frac{2}{4} = \frac{1}{2} = m$$

Equation of the normal is $y - y_1 = \frac{-1}{m}(x - x_1)$

Here $m = \frac{1}{2}$, $x_1 = 4$, $y_1 = 4$

$$y - 4 = \frac{-1}{\frac{1}{2}}(x - 4)$$

$$y - 4 = -2(x - 4)$$

$$y - 4 = -2x + 8$$

$$2x + y - 4 - 8 = 0$$

$$2x + y - 12 = 0$$

19.(c). Find the maximum and minimum of

$$y = 2x^3 + 3x^2 - 36x + 1$$

Answer Refer *October 2017 ,Question no:* 19(c) *,Page no:* 72

20.(a). Find the volume of a right circular cone of height 'h' and radius 'r' by integration

Answer Refer *April 2016 ,Question no:* 20(a) *,Page no:* 17

20.(b). Solve: $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

Answer

Given $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$$

Integrating on both sides, we get

$$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1} y = \sin^{-1} x + C$$

$$\sin^{-1} x - \sin^{-1} y = C$$

20.(c). Solve: $\frac{dy}{dx} + \frac{y}{x} = x^2$

Answer

Given $\frac{dy}{dx} + \frac{y}{x} = x^2$

This is of the form $\frac{dy}{dx} + Py = Q$

Here, $P = \frac{1}{x}$; $Q = x^2$

Integrating Factor = $e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$

∴, The solution is $y e^{\int P dx} = \int Q\, e^{\int P dx} dx + C$

$$y\,x = \int x^2 (x)\, dx + C$$

$$= \int x^3\, dx + C$$

$$= \frac{x^4}{4} + C$$

21.(a). Solve: $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

Answer

Given $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

$$(D^2 - 6D + 8)y = 0$$

Auxiliary equation is $m^2 - 6m + 8 = 0$

$$(m - 2)(m - 4) = 0$$

$(m - 2) = 0$ | $(m - 4) = 0$

$m = 2$ | $m = 4$

∴ The solution is $y = Ae^{m_1 x} + Be^{m_2 x}$

$$y = Ae^{2x} + Be^{4x}$$

21.(b). Solve: $(D^2 + 3D + 2)y = 5e^{-4x}$

Answer

Given, $(D^2 + 3D + 2)y = 5e^{-4x}$

Auxiliary Equation is $m^2 + 3m + 2 = 0$

$$(m + 1)(m + 2) = 0$$

$m + 1 = 0$ | $m + 2 = 0$

$m = -1$ | $m = -2$

∴, Complementary Function $(C.F) = Ae^{-x} + Be^{-2x}$

Particular Integral = $\frac{e^{ax}}{f(D)}$

$$P.I. = \frac{5e^{-4x}}{D^2 + 3D + 2}, \text{ Replace } D \text{ by } -4$$

$$P.I. = \frac{5e^{-4x}}{(-4)^2 + 3(-4) + 2}$$

$$P.I. = \frac{5e^{-4x}}{16 - 12 + 2}$$

$$P.I. = \frac{5e^{-4x}}{6}$$

General Solution, $\boldsymbol{y = C.F + P.I}$

$$y = Ae^{-x} + Be^{-2x} + \frac{5e^{-4x}}{6}$$

21.(c). Solve: $(D^2 + 9)y = \cos 2x$

Answer Given $(D^2 + 9)y = \cos 2x$

Auxiliary equation is $m^2 + 9 = 0$

$$m^2 = -9$$

$$m = \pm\sqrt{-9} = \pm i3$$

Here, $\alpha = 0, \ \beta = 3$

∴Complementary Function is $e^{0x}[A\cos 3x + B\sin 3x]$

$$C.F. = A\cos 3x + B\sin 3x$$

$$Particular\ Integral = \frac{\cos ax}{f(D)}$$

$$= \frac{\cos 2x}{D^2 + 9}$$

Replace $D^2 = -2^2 = -4$

$$= \frac{\cos 2x}{-4 + 9} = \frac{\cos 2x}{5}$$

General solution is $y = C.F + P.I$

$$\therefore y = A\cos 3x + B\sin 3x + \frac{\cos 2x}{5}$$