APRIL 2017

Time – Three hours

(*Maximum Marks*: 75)

[N.B:- (1) Answer any FIVE questions in each of PART-A& PART-B and any two divisions of each questions in PART-C

(2) Each questions carries 2(two)marks in PART-A ,3(three)marks in PART-B and 5(five)marks for each division in PART-C.]

PART – A

1. Define random variable
2. If $E(X) = 5$ and $E(X^2) = 35$, find the variance of 'X'
3. Give two examples of poisson distribution
4. If Z is the standard normal variate write down the values of mean and standard deviation of the distribution
5. Find the slope of tangent to the curve $y = x^2$ at (1, 1)
6. Write down the condition for maximum value of $y = f(x)$ at x = a
7. Solve: $xdx = ydy$
8. Find the auxiliary equation of $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

PART – B

9. If a random variable 'X' has the following probability distribution , find *E(X)*

X	0	1	2	3

$P(X)$	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{3}{7}$

10. A binomial distribution has mean 4 and variance $\frac{8}{3}$, find'p' and'n'
11. If the variance of a Poisson distribution is 0.25 ,find $P(X = 0)$
12. Write down the normal equations to fit a straight line $y = ax + b$
13. The distance 's' travelled by a particle in't' secs is given by $s = 4t^3 - 5t + 6$. Find its velocity and acceleration after 10 secs
14. Find the minimum value of $y = x^2 - 4x$
15. Find the integrating factor of $\frac{dy}{dx} + \frac{2}{x}y = x$
16. Solve: $(D^2 - 25)y = 0$

PART- C

17.(a) If a random variable 'X' has the following probability distribution, find (i) the value of 'a'(ii) $P(X > 3)$ (iii) $P(1 < X < 4)$

X	0	1	2	3	4	5
$P(X)$	a	3a	5a	7a	9a	11a

(b) Show that $f(x) = \begin{cases} \frac{1}{9}x^2, 0 < x < 3 \\ 0, \text{ otherwise} \end{cases}$

$is\ a$ probability density function

(c) Ten coins are tossed simultaneously. Find the probability of getting at least 7 heads

18.(a) If 2% of electric bulbs produced by a company are defective, find the probability that in a sample of 200 bulbs, exactly 4 bulbs are defective.[$e^{-4} = 0.0183$]

(b) If $'X'$ is normally distributed with mean 6 and standard deviation 5,find(i) $P(-4 < X < 16)$(ii)$P(X > 11)$

Given that $P(0 < z < 2) = 0.4772, P(0 < z < 1) = 0.3413$

(c) Fit a straight line for the data given below

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

19.(a) The distance 's' traveled by a particle in 't' secs is given by $s = ae^{t} + be^{-t}$ .Prove that the acceleration is always equal to the distance passed over

(b) Find the equation to the tangent and normal to the curve $y = 6 + x - x^2$ at (2 , 4)

(c) For $y = 2x^3 - 15x^2 - 36x + 18$, find the maximum and minimum values

20.(a) Find the volume of a sphere of radius r units using Integration

(b) Solve:$(1 + e^x)sec^2y\ dy - e^x tan\ y\ dx = 0$

(c) Solve: $\frac{dy}{dx} + y \tan x = e^{x} \cos x$

21.(a) Solve: $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$

(b) Solve: $(D^2 - 5D + 6)y = e^{4x}$

(c) Solve: $(D^2 + 16)y = \sin 9x$

ANSWERS

PART - A

1. Define random variable

Answer The function X: S → R is a random variable where S is sample space and R is real line

2.If $E(X) = 5$ and $E(X^2) = 35$, find the variance of 'X'

Answer Variance = $E(X^2) - [E(X)]^2$

$= 35 - (5)^2$

$= 35 - 25$

$= 10$

3. Give two examples of poisson distribution?

Answer (i) Goals scored in a football match

(ii) Number of air accidents per year in a city

4. If Z is the standard normal variate, write down the values of mean and standard deviation of the distribution

Answer Refer *April 2016 ,Question no: 4 ,Page no: 5*

5. Find the slope of the tangent to the curve $y = x^2$ at (1, 1)

Answer

$$y = x^2$$

$$\frac{dy}{dx} = 2x$$

$$\left(\frac{dy}{dx}\right)_{(1,1)} = 2(1) = 2$$

$\therefore$ Slope of the tangent $m = 2$

6. Write the condition for maximum of the function $y = f(x)$ at $x = a$

Answer Condition for maximum.

(i) $\frac{dy}{dx} = 0$ at $x = a$

(ii) $\frac{d^2y}{dx^2} = -ve \ (or) < 0$ at $x = a$

7. Solve: $xdx = ydy$

Answer Given $xdx = ydy$

$$\int x\,dx = \int y\,dy$$

$$\frac{x^2}{2} = \frac{y^2}{2} + C$$

8. Find the auxiliary equation of $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

Answer Auxiliary equation is $m^2 + 2m + 1 = 0$

PART - B

9. If a random variable 'X' has the following probability distribution , find *E(X)* .

X	0	1	2	3
$P(X)$	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{3}{7}$

Answer

Formula: $E(X) = \sum_{i=1}^{n} x_i p_i$

$$E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n$$

$$= \left(0 \times \frac{1}{7}\right) + \left(1 \times \frac{2}{7}\right) + \left(2 \times \frac{1}{7}\right) + \left(3 \times \frac{3}{7}\right)$$

$$= 0 + \frac{2}{7} + \frac{2}{7} + \frac{9}{7}$$

$$= \frac{2+2+9}{7} = \frac{13}{7}$$

10. A binomial distribution has mean 4 and variance $\frac{8}{3}$, find 'p' and 'n'

Answer Given variance $npq = \frac{8}{3}$ – – – (1)

Mean $np = 4$ – – – (2)

$$\frac{(1)}{(2)} \Rightarrow \frac{npq}{np} = \frac{\frac{8}{3}}{4}$$

$$\boldsymbol{q} = \frac{2}{3}$$

Since $\boldsymbol{p} = 1 - q$

$$= 1 - \frac{2}{3} = \frac{3-2}{3} = \frac{1}{3}$$

$$(2) \Rightarrow np = 4$$

$$n\left(\frac{1}{3}\right) = 4$$

$$\Rightarrow \boldsymbol{n} = \mathbf{12}$$

11. If the variance of a poisson distribution is 0.25 ,find $P(X = 0)$

Answer

Formula: $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$,

Given: $variance\ \lambda = 0.25$

$$P(X = x) = \frac{e^{-0.25}(0.25)^x}{x!}$$

$$P(X = 0) = \frac{e^{-0.25}(0.25)^0}{0!} = e^{-0.25}$$

12. Write down the normal equation to fit a straight line $y = ax + b$

Answer

Normal Equation are

$$a\sum x + nb = \sum y$$

$$a\sum x^2 + b\sum x = \sum xy$$

13. The distance 's' travelled by a particle in '*t*' secs is given by s = $4t^3 - 5t + 6$ .Find its velocity and acceleration after 10 secs

Answer

$s = 4t^3 - 5t + 6$

$$v = \frac{ds}{dt} = 4(3t^2) - 5(1) + 0 = 12t^2 - 5$$

$$a = \frac{d^2s}{dt^2} = 12(2t) - 0 = 24t$$

When t = 10 seconds

$$v = \left(\frac{ds}{dt}\right)_{t=10} = 12(10)^2 - 5 = 1195 \text{ units / sec}$$

$$a = \left(\frac{d^2s}{dt^2}\right)_{t\,=\,10} = 24(10) = 240 \text{ units /sec}^2$$

14. Find the minimum value of $y = x^2 - 4x$

Answer Refer April 2016, Question no: 12, Page no: 7

15. Find the integrating factor of $\frac{dy}{dx} + \frac{2}{x}y = x$

Answer Given $\frac{dy}{dx} + \frac{2}{x}y = x$

This is of the form $\frac{dy}{dx} + Py = Q$

Here, $P = \frac{2}{x}; \quad Q = x$

Integrating Factor = $e^{\int P dx}$

$= e^{\int \frac{2}{x} dx}$

$= e^{2\int \frac{1}{x} dx}$

$= e^{2\log x} = e^{\log x^2} = x^2$

16. Solve: $(D^2 - 25)y = 0$

Answer Given $(D^2 - 25)y = 0$

Auxiliary equation is $m^2 - 25 = 0$

$m^2 = 25$

$m = \pm\sqrt{25} = \pm 5$

$\therefore$ The solution is $y = Ae^{m_1 x} + Be^{m_2 x}$

$y = Ae^{5x} + Be^{-5x}$

PART - C

17.(a). If a random variable '*X*' has the following probability

distribution, find (i) the value of 'a'(ii) $P\ (X > 3)$

(iii) $P\ (1 < X < 4)$

X	0	1	2	3	4	5
P(X)	a	3a	5a	7a	9a	11a

Answer

(i) We know that $\sum P_i = 1$

$$a + 3a + 5a + 7a + 9a + 11a = 1$$

$$36a = 1$$

$$\mathbf{a = \frac{1}{36}}$$

(ii) $P\ (X > 3) = P\ (X = 4) + P\ (X = 5)$

$$= 9a + 11a$$

$$= 20a$$

$$= 20\left(\frac{1}{36}\right)$$

$$= \frac{20}{36} = \frac{5}{9}$$

(iii) $P\ (1 < X < 4) = P\ (X = 2) + P\ (X = 3)$

$$= 5a + 7a$$

$$= 12a$$

$$= 12\left(\frac{1}{36}\right)$$

$$= \mathbf{\frac{12}{36}}$$

$$=\frac{1}{3}$$

17.(b). Show that f(x) = $\begin{cases} \frac{1}{9}x^2, 0 < x < 3 \\ 0, \quad elsewhere \end{cases}$

***is a* probability density function**

Answer

Given f(x) = $\frac{1}{9}x^2, 0 < x < 3$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^3 \frac{1}{9}x^2\,dx$$

$$= \frac{1}{9}\int_0^3 x^2\,dx$$

$$= \frac{1}{9}\left[\frac{x^3}{3}\right]_0^3$$

$$= \frac{1}{9}\left[\frac{3^3 - 0^3}{3}\right]$$

$$= \frac{1}{9}\left[\frac{27}{3}\right]$$

$$= \frac{27}{27}$$

$$= 1$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$\therefore f(x)$ is a probability density function.

17.(c). Ten coins are tossed simultaneously. Find the probability of getting at least 7 heads.

Answer Refer *October 2017,Question no:* 17(c) *,Page no:* 66

18.(a). If 2% of electric bulbs produced by a company are defective ,find the probability that in a sample of 200 bulbs, exactly 4 bulbs are defective.[$e^{-4} = 0.0183$]

Answer

We know that $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given p= $2\% = \frac{2}{100}$; $n = 200$

We know that $\lambda = np = 200\left(\frac{2}{100}\right) = 4$

$$P(X = x) = \frac{e^{-4}(4)^x}{x!}$$

Exactly 4 are defective: $\boldsymbol{P(X = 4)}$

$$P(X = 4) = \frac{e^{-4}(4)^4}{4!}$$

$$P(X = 4) = \frac{0.0183\ (256)}{24}$$

$$= 0.1952$$

18.(b). If $'X'$ is normally distributed with mean 6 and standard deviation 5,find(i) $P(-4 < X < 16)$(ii)$P(X > 11)$. Given that $P(0 < z < 2) = 0.4772, P(0 < z < 1) = 0.3413$

Answer

Given, Mean = 6 ,

Standard Deviation $\sigma = 5$

We know that $z = \frac{X-\mu}{\sigma} = \frac{X-6}{5}$

(i) $P(-4 < X < 16)$

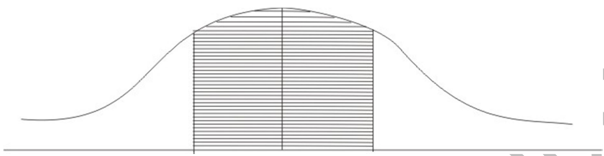
When $X = -4$ | When $X = 16$

$$z = \frac{-4-6}{5} \qquad\qquad z = \frac{16-6}{5}$$

$$z = \frac{-10}{5} \qquad\qquad z = \frac{10}{5}$$

$$z = -2 \qquad\qquad z = 2$$

$\therefore \boldsymbol{P(-4} < X < 16) = \boldsymbol{P(-2} < z < 2)$



$= P(-2 < z < 0) + P(0 < z < 2)$

$= P(0 < z < 2) + P(0 < z < 2)$

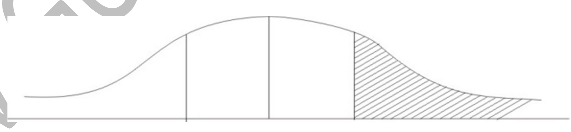
$= 0.4772 + 0.4772$

$= \mathbf{0.9544}$

(ii) $\boldsymbol{P}(\boldsymbol{X} > 11)$, when $\boldsymbol{X} = \mathbf{11}$

$$z = \frac{11-6}{5} = \frac{5}{5} = 1$$

$\therefore \boldsymbol{P}(\boldsymbol{X} > 11) = \boldsymbol{P}(\boldsymbol{z} > 1)$



$= P(0 < z < \infty) - P(0 < z < 1)$

$= 0.5 - 0.3413$

$\boldsymbol{P}(\boldsymbol{X} > 11) = \mathbf{0.1587}$

18.(c) Fit a straight line for the data given below.

x	0	1	2	3	4

y	**1**	**1.8**	**3.3**	**4.5**	**6.3**

Answer

Let $y = ax + b$ ... (1) be the line of best fit.

Then the normal equations are

$$a\sum x_i + nb = \sum y_i \quad .... \quad \textbf{(2)}$$

$$a\sum x_i{}^2 + b\sum x_i = \sum x_i y_i \quad .... \quad \textbf{(3)}$$

We compute $\sum xi$, $\sum xi^2$, $\sum yi$ and $\sum xi\, yi$ from the following table.

xi	yi	xi^2	$xiyi$
0	1	0	0
1	1.8	1	1.8
2	3.3	4	6.6
3	4.5	9	13.5
4	6.3	16	25.2
$\sum \boldsymbol{x_i} = 10$	$\sum \boldsymbol{y_i} = 16.9$	$\sum \boldsymbol{x_i^2} = 30$	$\sum \boldsymbol{x_i y_i} = 47.1$

Here, $n = 5$

Using the normal equations, we get

(2)⟹ $$\boldsymbol{a\sum x_i + nb = \sum y_i}$$

$$a(10) + (5)b = 16.9$$

$$10a + 5b = 16.9$$

(3)⟹ $$a\sum x_i{}^2 + b\sum x_i = \sum x_i y_i$$

$$a(30) + b(10) = 47.1$$

$$30a + 10b = 47.1$$

By Cramer's Rule :

$10a + 5b = 16.9$ and $30a + 10b = 47.1$

$$\Delta = \begin{vmatrix} 10 & 5 \\ 30 & 10 \end{vmatrix} = 100 - 150 = -50$$

$$\Delta_a = \begin{vmatrix} 16.9 & 5 \\ 47.1 & 10 \end{vmatrix} = 169 - 235.5 = -66.5$$

$$\Delta_b = \begin{vmatrix} 10 & 16.9 \\ 30 & 47.1 \end{vmatrix} = 471 - 507 = -36$$

$$a = \frac{\Delta_a}{\Delta} = \frac{-66.5}{-50} = \mathbf{1.33}$$

$$b = \frac{\Delta_b}{\Delta} = \frac{-36}{-50} = \mathbf{0.72}$$

(1) $\Rightarrow y = ax + b$

Put $a = 1.33$ and $b = 0.72$

$\boldsymbol{y = 1.33x + 0.72}$, which is the line of best fit.

19.(a). The distance 's' travelled by a particle in 't' secs is given by $s = ae^{t} + be^{-t}$ .Prove that the acceleration is always equal to the distance passed over

Answer Refer *April 2016 ,Question no:* 11 *,Page no:* 6

19.(b). Find the equation to the tangent and normal to the curve $y = 6 + x - x^2$ at (2 , 4)

Answer Refer *October 2017,Question no:* 19(b) *,Page no:* 71

19.(c). For $y = 2x^3 - 15x^2 - 36x + 18$,find the maximum and minimum values

Answer Refer *April 2016 ,Question no:* 19(c) *,Page no:* 16

20.(a). Find the volume of a sphere of radius 'r' units using integration

Answer Refer *October 2017,Question no:* 20(a) *,Page no:* 73

20.(b). Solve: $\boldsymbol{(1 + e^x)sec^2y\ dy - e^x tan\, y\, dx = 0}$

Answer Refer *October 2017, Question no:* 20(b) *, Page no:* 74

20.(c). Solve: $\frac{dy}{dx} + y\tan x = e^x \cos x$

Answer Given $\frac{dy}{dx} + y\tan x = e^x \cos x$

This is of the form $\frac{dy}{dx} + Py = Q$

Here $P = \tan x$; $Q = e^x \cos x$

Integrating Factor $= e^{\int P dx}$

$= e^{\int \tan x \, dx}$

$= e^{\log \sec x}$

$= \sec x$

∴, The solution is $y e^{\int P dx} = \int Q e^{\int P dx} dx + C$

$$y \sec x = \int e^x \cos x \sec x dx$$

$$= \int e^x \cos x \frac{1}{\cos x} dx + C$$

$$= \int e^x dx + C$$

$$= e^x + C$$

21.(a). Solve: $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$

Answer

Given $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$

$$(D^2 + D + 1)y = 0$$

Auxiliary equation is $m^2 + m + 1 = 0$

We know, $m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Here $a = 1,\ b = 1,\ c = 1$

$$m = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm i\sqrt{3}}{2}$$

$$= \frac{-1}{2} \pm \frac{i\sqrt{3}}{2} \quad (\alpha \pm i\beta)$$

$$Here\ \alpha = -\frac{1}{2},\quad \beta = \frac{\sqrt{3}}{2}$$

∴, The solution is $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$$y = e^{-\frac{1}{2}x}\left(A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x\right)$$

21.(b). Solve: $(D^2 - 5D + 6)y = e^{4x}$

Answer Given, $(D^2 - 5D + 6)y = e^{4x}$

Auxiliary Equation is $m^2 - 5m + 6 = 0$

$$(m-2)(m-3) = 0$$

$m - 2 = 0 \quad | \quad m - 3 = 0$

$m = 2 \quad | \quad m = 3$

∴, Complementary Function $(C.F) = Ae^{2x} + Be^{3x}$

Particular Integral = $\frac{e^{ax}}{f(D)}$

$$P.I. = \frac{e^{4x}}{D^2 - 5D + 6}$$

Replace D by 4

$$P.I. = \frac{e^{4x}}{(4)^2 - 5(4) + 6}$$

$$P.I. = \frac{e^{4x}}{16 - 20 + 6}$$

$$P.I. = \frac{e^{4x}}{2}$$

General Solution, $y = C.F + P.I$

$$y = Ae^{2x} + Be^{3x} + \frac{e^{4x}}{2}$$

21.(c). Solve: $\boldsymbol{(D^2 + 16)y = \sin 9x}$

Answer Refer *April 2016 ,Question no:* 21(c) *,Page no:* 21

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