APRIL 2016

$Time$ – $Three$ hours

($Maximum\ Marks$: 75)

[N.B:- (1) Answer any FIVE questions in each of PART-A& PART-B and any two divisions of each questions in PART-C

(2) Each questions carries 2(two)marks in PART-A ,3(three)marks in PART-B and 5(five)marks for each division in PART-C.]

PART – A

1. If f(x) is a probability density function then what is the value of $\int_{-\infty}^{\infty} f\ (x)dx$?
2. If the mean and variance of a binomial distribution are 12 and 6 , find 'p'
3. If a random variable 'X' follows Poisson distribution such that $P(X = 1) = P(X = 2)$, find the mean
4. Write down the mean and standard deviation of the standard normal distribution
5. If $s = 5t^2 + 6t + 5$, find the initial velocity
6. Find the slope of normal to the curve $y = x^3$ at (4, –2)
7. State the order and degree of $\left(\frac{dy}{dx}\right)^2 + 7\ \frac{d^2y}{dx^2} + 2y = 0$
8. Solve: $(D^2 - 49)y = 0$

PART – B

9. If a random variable 'X' has the following probability distribution , find E(X)

X	1	2	3
$P(X)$	$\frac{1}{2}$	0	$\frac{1}{2}$

10. Mention any three properties of normal curve

11. If $x = ae^{t} + be^{-t}$. Show that the acceleration is always equal to the distance passed over

12. Find the minimum value of $y = x^2 - 4x$

13. Solve: $x\,dx + y\,dy = 0$

14. Find the integrating factor of $\frac{dy}{dx} + \frac{1}{x}y = x$

15. Solve: $(D^2 - 5D + 6)y = 0$

16. Find the particular integral of $(D^2 - 10D + 1)y = e^{-x}$

PART- C

17. (a) A random variable 'X' has the following probability distribution

X	0	1	2	3
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$

Find (i) $E(X)$ and (ii) $E(X^2)$

(b) A random variable 'X' has the following probability distribution

X	0	1	2	3	4
P (X = x)	a	3a	5a	7a	9a

Find (i) 'a' and (ii) $P(X \geq 2)$

(c) In a binomial distribution, if n = 15 and $P(X = 1) = 3P(X = 0)$, find the value of 'p'

18. (a) If 3% of the electric bulbs are defective, find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective . ($e^{-3} = 0.0498$)

(b) In a normal distribution mean is 10 and standard deviation is 3 . Find the probability interval from $X = 8.6$ to $X = 12.8$

(c) Fit a straight line for the following data

x	0	1	2	3	4
y	10	14	19	26	31

19 (a) If the distance travelled equation of a particle is given by s = a cos 6t + b sin 6t . Show that the acceleration varies as its distance

(b) Find the equation to the tangents to the curve $y = x^2 + x - 6$ at the point where it cuts the x – axis

(c) Find the maximum and minimum values of $y = 2x^3 - 15x^2 - 36x + 18$

20.(a) Find the volume of a right circular cone of base radius $'r'$ and height '⍰' by using integration

(b) Solve: $\tan x \sec^2 y \; dy + \tan y \sec^2 x \; dx = 0$

(c) Solve: $\frac{dy}{dx} - \frac{2y}{x} = x^2 \sin x$

21. (a) Solve: $(D^2 + D - 2)y = 0$

(b) Solve: $(D^2 - 8D + 16)y = 2e^x$

(c) Solve: $(D^2 + 16)y = \sin 9x$

ANSWERS

PART - A

1. If f(x) is a probability density function then what is the value of $\int_{-\infty}^{\infty} f(x)dx$?

Answer The value of $\int_{-\infty}^{\infty} f(x)dx = 1$

2. If the mean and variance of a binomial distribution are 12 and 6 , find 'p'

Answer Given: Mean = $np = 12$ - - - - - (1)

Variance = $npq = 6$ - - - - - (2)

$$\frac{(2)}{(1)} \Rightarrow \frac{npq}{np} = \frac{6}{12}$$

$$\Rightarrow q = \frac{1}{2}$$

$$\therefore \boldsymbol{p} = 1 - q = 1 - \frac{1}{2} = \boldsymbol{\frac{1}{2}}$$

3. If a random variable 'X' follows poisson distribution such that $P(X = 1) = P(X = 2)$, find mean

Answer Formula: $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given $P(X = 1) = P(X = 2)$

$$\frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

$$\frac{\lambda}{1} = \frac{\lambda^2}{2}$$

$$\frac{2}{1} = \frac{\lambda^2}{\lambda} \Rightarrow \lambda = 2$$

$\therefore$Mean = 2

4. Write down the mean and standard deviation of the standard normal distribution

Answer

Mean $\mu = 0$

Standard Deviation $\sigma = 1$

5. If s = $5t^2 + 6t + 5$, find the initial velocity

Answer Given: $s = 5t^2 + 6t + 5$

$$v = \frac{ds}{dt} = 5(2t) + 6(1) + 0 = 10t + 6$$

Initial velocity $v = \left(\frac{ds}{dt}\right)_{t=0} = 10(0) + 6 = 6$ units / sec

6. Find the slope of the normal to the curve $y = x^3$ at (4, –2)

Answer Given: $y = x^3$

$$\frac{dy}{dx} = 3x^2$$

$$\left(\frac{dy}{dx}\right)_{(4,-2)} = 3(4)^2 = 48$$

Slope of the tangent, $m = 48$

$\therefore$ Slope of the normal, $\frac{-1}{m} = \frac{-1}{48}$

7. State the order and degree of $\left(\frac{dy}{dx}\right)^2 + 7\frac{d^2y}{dx^2} + 2y = 0$

Answer Order = 2 and Degree = 1

8. Solve: $(D^2 - 49)y = 0$

Answer Given $(D^2 - 49)y = 0$

Auxiliary equation is $m^2 - 49 = 0$

$$m^2 = 49$$

$$m = \pm\sqrt{49} = \pm 7$$

$\therefore$ The solution is $y = Ae^{m_1 x} + Be^{m_2 x}$

$$y = Ae^{7x} + Be^{-7x}$$

PART - B

9. If a random variable 'X' has the following probability distribution , find E(X)

X	1	2	3
P (*X*)	$\frac{1}{2}$	0	$\frac{1}{2}$

Answer

Formula: $E(X) = \sum_{i=1}^{n} x_i p_i$

$$E(X) = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$$

$$= \left(1 \times \frac{1}{2}\right) + (2 \times 0) + \left(3 \times \frac{1}{2}\right)$$

$$= \frac{1}{2} + 0 + \frac{3}{2}$$

$$= \frac{1+3}{2}$$

$$= \frac{4}{2}$$

$$= 2$$

10. Mention any three properties of normal curve

Answer

(i).The normal curve is bell shaped

(ii). It is symmetrical about the line $X = \mu$

(iii). Mean = Median = Mode = μ

11. If $s = ae^t + be^{-t}$. Show that the acceleration is always equal to the distance passed over

Answer

Given: $s = ae^t + be^{-t}$

$$v = \frac{ds}{dt} = ae^{t} - be^{-t} \quad \{ \text{since } \frac{d}{dt}(e^{-t}) = -e^{-t} \}$$

$$a = \frac{d^2s}{dt^2} = ae^{t} + be^{-t}$$

$$\Rightarrow a = s$$

$\therefore$ Acceleration is always equal to the distance passed over

12. Find the minimum value of $y = x^2 - 4x$

Answer

Given: $y = x^2 - 4x$

$$y_1 = 2x - 4$$

$$y_2 = 2$$

Put $y_1 = 0 \Rightarrow 2x - 4 = 0$

$$2x = 4$$

$$\Rightarrow x = 2$$

Now $(y_2)_{x=2} = 2 > 0$

$\therefore y$ is minimum at $x = 2$

The minimum value of $y = (2)^2 - 4(2)$

$$= 4 - 8 = -4$$

13. Solve: $xdx + ydy = 0$

Answer

Given $xdx + ydy = 0$

$$xdx = -ydy$$

$$\int x\,dx = -\int y\,dy$$

$$\frac{x^2}{2} = -\frac{y^2}{2} + C$$

$$\frac{x^2}{2} + \frac{y^2}{2} = C$$

14. Find the integrating factor of $\frac{dy}{dx} + \frac{1}{x}y = x$

Answer

Given $\frac{dy}{dx} + \frac{1}{x}y = x$

This is of the form $\frac{dy}{dx} + Py = Q$

Here, $P = \frac{1}{x}$; $Q = x$

Integrating Factor $= e^{\int P dx}$

$= e^{\int \frac{1}{x} dx}$

$= e^{\log x} = x$

15. Solve: $(D^2 - 5D + 6)y = 0$

Answer

Given $(D^2 - 5D + 6)y = 0$

Auxiliary equation is $m^2 - 5m + 6 = 0$

$(m - 2)(m - 3) = 0$

$m - 2 = 0$ | $m - 3 = 0$

$m = 2$ | $m = 3$

$\therefore$The solution is $y = Ae^{m_1 x} + Be^{m_2 x}$

$y = Ae^{2x} + Be^{3x}$

16. Find the particular integral of

$(D^2 - 10D + 1)y = e^{-x}$

Answer

Given $(D^2 - 10D + 1)y = e^{-x}$

$$P.I. = \frac{e^{-x}}{D^2 - 10D + 1}$$

Replace D by -1

$$P.I. = \frac{e^{-x}}{(-1)^2 - 10(-1) + 1}$$

$$P.I. = \frac{e^{-x}}{1 + 10 + 1}$$

$$\therefore, \ P.I. = \frac{e^{-x}}{12}$$

PART - C

17.(a). A random variable 'X' has the following probability distribution

X	0	1	2	3
$P(X)$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$

Find (i) $E(X)$ and (ii) $E(X^2)$

Answer

(i) Formula: $E(X) = \sum_{i=1}^{n} x_i p_i$

$$E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n$$

$$= \left(0 \times \frac{1}{3}\right) + \left(1 \times \frac{1}{6}\right) + \left(2 \times \frac{1}{6}\right) + \left(3 \times \frac{1}{3}\right)$$

$$= 0 + \frac{1}{6} + \frac{2}{6} + 1$$

$$= \frac{1 + 2 + 6}{6}$$

$$= \frac{9}{6}$$

$$= \frac{3}{2}$$

(ii) Formula: $E(X^2) = \sum_{i=1}^{n} x_i^2 \ P_i$

$$E(X^2) = x_1^2p_1 + x_2^2p_2 + \cdots + x_n^2p_n$$

$$= \left(0^2 \times \frac{1}{3}\right) + \left(1^2 \times \frac{1}{6}\right) + \left(2^2 \times \frac{1}{6}\right) + \left(3^2 \times \frac{1}{3}\right)$$

$$= \left(\mathbf{0} \times \frac{1}{3}\right) + \left(\mathbf{1} \times \frac{1}{6}\right) + \left(\mathbf{4} \times \frac{1}{6}\right) + \left(\mathbf{9} \times \frac{1}{3}\right)$$

$= 0 + \frac{1}{6} + \frac{4}{6} + 3$

$= \frac{1+4+18}{6}$

$= \frac{23}{6}$

17.(b). A random variable 'X' has the following probability distribution

X	0	1	2	3	4
$P(X)$	a	3a	5a	7a	9a

Find (i) 'a' and (ii) $P(X \geq 2)$

Answer (i) We know that $\sum P_i = 1$

$a + 3a + 5a + 7a + 9a = 1$

$25a = 1 \Rightarrow \mathbf{a} = \frac{1}{25}$

(ii) $P(X \geq 2) = P(X=2) + P(X=3) + P(X=4)$

$= 5a + 7a + 9a$

$= 21a$

$= 21\left(\frac{1}{25}\right)$

$= \frac{21}{25}$

17.(c). In a binomial distribution, if n = 15 and $P(X = 1) = 3P(X = 0)$, find the value of 'p'

Answer Given $n = 15$

Binomial distribution is $\boldsymbol{P(X = x) = nc_x p^x q^{n-x}}$

$P(X = x) = 15c_x\, p^x q^{15-x}$

Given $\boldsymbol{P}\,(\boldsymbol{X} = \mathbf{1}) = \mathbf{3}\,\boldsymbol{P}(\boldsymbol{X} = \mathbf{0})$

$$15c_1\,p^1\,q^{15-1} = 3\;\;15c_0\,p^0\,q^{15-0}$$

$$15\,p\,q^{14} = 3 \times 1 \times 1 \times\; q^{15}$$

$$15\,p\,q^{14} = 3\,q^{15}$$

$$15\,p = 3q$$

$$15\,p = 3(1 - p) \quad [\therefore q = 1 - p]$$

$$15\,p = 3 - 3p$$

$$15p + 3p = 3$$

$$18\,p = 3$$

$$p = \frac{3}{18}$$

$$p = \frac{1}{6}$$

18.(a). If 3% of the electric bulbs are defective, find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective . ($\boldsymbol{e^{-3} = 0.0498}$)

Answer

Formula: $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given $P = 3\% = \frac{3}{100}$; $n = 100$

We know that $\lambda = nP = 100\left(\frac{3}{100}\right) = 3$

$$P(X = x) = \frac{e^{-3}(3)^x}{x!}$$

Exactly 5 are defective: $P(X = 5)$

$$P(X = 5) = \frac{e^{-3}(3)^5}{5!}$$

$$P(X = 5) = \frac{0.0498\,(243)}{120}$$

$$= 0.1008$$

18.(b). If a normal distribution mean is 10 and standard deviation is 3 . Find the probability interval from $X = 8.6$ to $X = 12.8$

Answer

Given, Mean $\mu = 10$

Standard Deviation $\sigma = 3$

We know that $z = \frac{X-\mu}{\sigma}$

$$= \frac{X - 10}{3}$$

Probability interval from $X = 8.6$ to $X = 12.8$

$\boldsymbol{P(8.6 < X < 12.8)}$

When $X = 8.6$

$$z = \frac{8.6 - 10}{3}$$

$$z = \frac{-1.4}{3}$$

$$z = -0.466 = -0.47$$

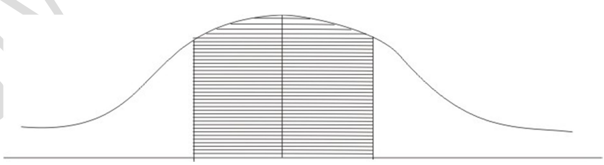
When $X = 12.8$

$$z = \frac{12.8 - 10}{3}$$

$$z = \frac{2.8}{3}$$

$$z = 0.933 = 0.93$$

$\therefore \boldsymbol{P(8.6 < X < 12.8) = P(-0.47} < z < 0.93)$



$-\infty$ $z = -0.47$ 0 $z = 0.93$ ∞

$= P(-0.47 < z < 0) + P(0 < z < 0.93)$

$= P(0 < z < 0.47) + P(0 < z < 0.93)$

$= 0.1808 + 0.3238$

$= \mathbf{0.5046}$

18.(c) Fit a straight line for the following data

x	0	1	2	3	4
y	10	14	19	26	31

Answer

Let $y = ax + b$ ... (1) be the line of best fit

Then the normal equations are

$$a\sum x_i + nb = \sum y_i \quad .... \quad \textbf{(2)}$$

$$a\sum x_i^2 + b\sum x_i = \sum x_i y_i \quad .... \quad \textbf{(3)}$$

We compute $\sum xi$, $\sum xi^2$, $\sum yi$ and $\sum xi\,yi$ from the following table.

xi	yi	xi^2	$xiyi$
0	10	0	0
1	14	1	14
2	19	4	38
3	26	9	78
4	31	16	124
$\sum \boldsymbol{x_i} = 10$	$\sum \boldsymbol{y_i} = 100$	$\sum \boldsymbol{x_i^2} = 30$	$\sum \boldsymbol{x_i y_i} = 254$

Here, $n = 5$

Using the normal equations, we get

(2)⟹ $\boldsymbol{a\sum x_i + nb = \sum y_i}$

$$a(10) + (5)b = 100$$

$$10a + 5b = 100$$

(3)⟹ $a\sum x_i^2 + b\sum x_i = \sum x_i y_i$

$$a(30) + b(10) = 254$$

$$30a + 10b = 254$$

By Cramer's Rule :

$10a + 5b = 100$ and $30a + 10b = 254$

$$\Delta = \begin{vmatrix} 10 & 5 \\ 30 & 10 \end{vmatrix} = 100 - 150 = -50$$

$$\Delta_a = \begin{vmatrix} 100 & 5 \\ 254 & 10 \end{vmatrix} = 1000 - 1270 = -270$$

$$\Delta_b = \begin{vmatrix} 10 & 100 \\ 30 & 254 \end{vmatrix} = 2540 - 3000 = -460$$

$$a = \frac{\Delta_a}{\Delta} = \frac{-270}{-50} = \mathbf{5.4}\ ,\quad b = \frac{\Delta_b}{\Delta} = \frac{-460}{-50} = \mathbf{9.2}$$

(1) $\Rightarrow y = ax + b$ Put $a = 5.4$ and $b = 9.2$

$\boldsymbol{y = 5.4\ x + 9.2}$, which is the line of best fit.

19.(a). If the distance travelled equation of a particle is given by s = a cos 6t + b sin 6t . Show that the acceleration varies as its distance

Answer

s = a cos 6t + b sin 6t —— (1)

$$v = \frac{ds}{dt}$$

v = a { (–sin6t) 6 } + b { (cos 6t) 6}

v = – 6a sin6t + 6b cos 6t

$$a = \frac{d^2s}{dt^2}$$

a = –6a { (cos6t) 6 } + 6b {(–sin 6t) 6 }

a = –36a cos6t – 36b sin6t

a = –36 { acos6t + bsin 6t }

a = –36 {s} [using (1)]

a = –36 × { Distance }

∴ acceleration varies as its distance.

19.(b). Find the equation of the tangents to the curve $y = x^2 + x - 6$ at the point where it cuts the x – axis

Answer

$$y = x^2 + x - 6$$

$$\frac{dy}{dx} = 2x + 1$$

The curve $y = x^2 + x - 6$ cuts x – axis. Put y = 0

$$x^2 + x - 6 = 0$$
$$(x - 2)(x + 3) = 0$$
$$x - 2 = 0 \quad | \quad x + 3 = 0$$
$$x = 2 \quad | \quad x = -3$$
$$\therefore (2, 0) \text{ and } (-3, 0)$$

(i) Slope of the tangent at (2, 0)

$$\left(\frac{dy}{dx}\right)_{(2,0)} = 2(2) + 1 = 5 = m$$

Equation of the tangent at (2, 0) is $y - y_1 = m(x - x_1)$

Here $m = 5$, $x_1 = 2$, $y_1 = 0$

$$y - 0 = 5(x - 2)$$

$$y = 5x - 10$$

$$5x - y - 10 = 0$$

(ii) Slope of the tangent at (–3, 0)

$$\left(\frac{dy}{dx}\right)_{(-3,0)} = 2(-3) + 1 = -5 = m$$

Equation of the tangent at (–3, 0) is $y - y_1 = m(x - x_1)$

Here $m = -5$, $x_1 = -3$, $y_1 = 0$

$$y - 0 = -5(x - (-3))$$

$$y = -5(x + 3)$$

$$y = -5x - 15$$

$$5x + y + 15 = 0$$

19.(c). Find the maximum and minimum values of

$$y = 2x^3 - 15x^2 - 36x + 18$$

Answer

$y = 2x^3 - 15x^2 - 36x + 18$

$y_1 = 2(3x^2) - 15(2x) - 36(1)+0$

$y_1 = 6x^2 - 30x - 36$

$y_2 = 6(2x) - 30(1) - 0$

$y_2 = 12x - 30$

Put $y_1 = 0 \Rightarrow 6x^2 - 30x - 36 = 0$

On ÷ by 6, we get $x^2 - 5x - 6 = 0$

$(x + 1)(x - 6) = 0$

$x + 1 = 0$	$x - 6 = 0$
$\mathbf{x = -1}$	$\mathbf{x = 6}$

Case (i) : When $x = -1$

Now $[y_2]_{x=-1} = 12(-1) - 30$

$= -12 - 30 = -42 < 0$

$\therefore y$ is maximum at $x = -1$

The maximum value of $y = 2(-1)^3 - 15(-1)^2 - 36(-1) + 18$

$= -2 - 15 + 36 + 18$

$= 37$

Case (ii): When $x = 6$

Now $[y_2]_{x=6} = 12(6) - 30$

$= 72 - 30 = 42 > 0$

$\therefore y$ is minimum at $x = 6$

The minimum value of $y = 2(6)^3 - 15(6)^2 - 36(6) + 18$

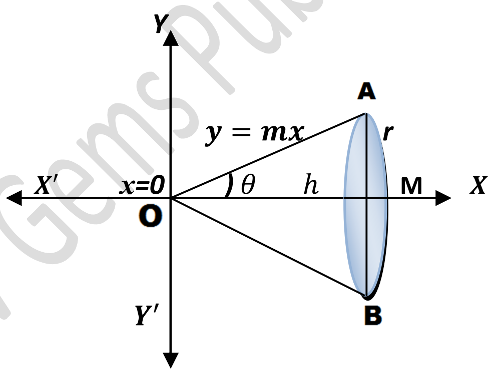
$= 2(216) - 15(36) - 216 + 18$

$= 432 - 540 - 216 + 18$

$= -306$

20.(a). Find the volume of a right circular cone of base radius $'r'$ and height $'h'$ by integration

Answer A right circular cone is formed by rotating a right angled triangle about x-axis.



The limits are $x = 0$ and $x = h$

$\therefore \quad a = 0 \text{ and } b = h$

Equation of the line OA is $y = mx$ ……. (1)

In $\triangle OAM$, $\tan\theta = \dfrac{opp.side}{adj.side}$

$$\tan\theta = \frac{r}{\boxed{?}}$$

$$\Rightarrow \quad m = \frac{r}{h} \qquad \text{since } m = \tan\theta$$

$$(1) \Rightarrow \quad y = \left[\frac{r}{\boxed{?}}\right] x$$

$$Volume\ of\ cone = \pi \int_a^b y^2\, dx$$

$$= \pi \int_0^h \left[\frac{rx}{\boxed{?}}\right]^2 dx$$

$$= \pi \int_0^h \frac{r^2x^2}{\boxed{?}^2} dx$$

$$= \frac{\pi r^2}{\boxed{?}^2} \int_0^h x^2\, dx$$

$$= \frac{\pi r^2}{\boxed{?}^2} \left[\frac{x^3}{3}\right]_0^h$$

$$= \frac{\pi r^2}{\boxed{?}^2} \left(\frac{\boxed{?}^3}{3}\right)$$

$$= \frac{\pi r^2 h}{3}\ cubic\ units$$

20.(b). Solve: $\boldsymbol{tan\, x\, sec^2 y\ \ dy + tan\, y\, sec^2 x\ \ dx = 0}$

Answer

Given $tan\, x\, sec^2 y\ \ dy + tan\, y\, sec^2 x\ \ dx = 0$

$$\tan x \sec^2 y\, dy = -\tan y \sec^2 x\, dx$$

$$\frac{\sec^2 y}{\tan y}dy = -\frac{\sec^2 x}{\tan x}dx$$

$$\int \frac{\sec^2 y}{\tan y}dy = -\int \frac{\sec^2 x}{\tan x}dx$$

$$\log(\tan y) = -\log(\tan x) + \log C$$

$$\log(\tan y) + \log(\tan x) = \log C$$

$$\log(\tan y \tan x) = \log C$$

$$\tan y \tan x = C$$

20.(c). Solve: $\frac{dy}{dx} - \frac{2y}{x} = x^2 \sin x$

Answer

Given $\frac{dy}{dx} - \frac{2y}{x} = x^2 \sin x$

This is of the form $\frac{dy}{dx} + Py = Q$

Here $P = -\frac{2}{x}$, $Q = x^2 \sin x$

Integrating Factor = $e^{\int P dx}$

$$= e^{\int \frac{-2}{x}dx}$$

$$= e^{-2\int \frac{1}{x}dx}$$

$$= e^{-2\log x}$$

$$= e^{\log x^{-2}} = x^{-2} = \frac{1}{x^2}$$

∴, The solution is $ye^{\int P dx} = \int Q e^{\int P dx} dx + C$

$$y \frac{1}{x^2} = \int x^2 \sin x \left(\frac{1}{x^2}\right) dx + C$$

$$= \int \sin x \, dx + C$$

$$= -\cos x + C$$

21.(a). Solve: $(D^2 + D - 2)y = 0$

Answer

Given $(D^2 + D - 2)y = 0$

Auxiliary equation is $m^2 + m - 2 = 0$

$$(m - 1)(m + 2) = 0$$

$(m - 1) = 0$	$(m + 2) = 0$
$m = 1$	$m = -2$

∴ The solution is $y = Ae^{m_1 x} + Be^{m_2 x}$

$$y = Ae^{x} + Be^{-2x}$$

21.(b). Solve: $(D^2 - 8D + 16)y = 2e^x$

Answer

Given, $(D^2 - 8D + 16)y = 2e^x$

Auxiliary Equation is $m^2 - 8m + 16 = 0$

$$(m - 4)(m - 4) = 0$$

$m - 4 = 0$	$m - 4 = 0$
$m = 4$	$m = 4$

∴, Complementary Function $= [\boldsymbol{Ax} + \boldsymbol{B}]\, \boldsymbol{e^{mx}}$

$$(C.F) = (Ax + B)e^{4x}$$

Particular Integral = $\frac{e^{ax}}{f(D)}$

$$P.I. = \frac{2e^x}{D^2 - 8D + 16}$$

Replace D by 1

$$P.I. = \frac{2e^x}{(1)^2 - 8(1) + 16}$$

$$P.I. = \frac{2e^x}{1 - 8 + 16}$$

$$P.I. = \frac{2e^x}{9}$$

General Solution, $y = C.F + P.I$

$$y = (Ax + B)e^{4x} + \frac{2e^x}{9}$$

21.(c). Solve: $\boldsymbol{(D^2 + 16)y = sin\ 9x}$

Answer Given $(D^2 + 16)y = sin\ 9x$

Auxiliary equation is $m^2 + 16 = 0$

$$m^2 = -16$$

$$m = \pm\sqrt{-16}$$

$$m = \pm i4$$

Here, $\alpha = 0, \quad \beta = 4$

∴Complementary function $= \boldsymbol{e^{\alpha x}[A\cos\beta x + B\sin\beta x}$)]

$$C.F = e^{0x}[Acos\ 4x + Bsin\ 4x] = Acos\ 4x + Bsin\ 4x$$

$$Particular\ Integral = \frac{\sin ax}{f(D)}$$

$$= \frac{\sin 9x}{D^2 + 16}$$

Replace $D^2 = -[9]^2 = -81$

$$= \frac{\sin 9x}{-81 + 16}$$

$$= \frac{\sin 9x}{-65}$$

General solution is $y = C.F + P.I$

$$\therefore y = Acos\ 4x + Bsin\ 4x - \frac{\sin 9x}{65}$$