OCTOBER 2017

Time – Three hours

(Maximum Marks: 75)

[N.B:- (1) Answer any FIVE questions in each of PART-A& PART-B and any two divisions of each questions in PART-C

(2) Each questions carries 2(two)marks in PART-A ,3(three)marks in PART-B and 5(five)marks for each division in PART-C.]

PART – A

1. Find the equation of the circle with centre is (1,– 2) and radius 5 units

2. Show that the circles $x^2 + y^2 - 4x + 2y + 5 = 0$ and $x^2 + y^2 - 4x + 2y - 8 = 0$ are concentric circles

3. If $2\vec{i} - \vec{j} + 3\vec{k}$ and $5\vec{i} + \vec{j} - 2\vec{k}$ *are* the position vectors of the points A and B , find $\overrightarrow{AB}$ and its modulus $|\overrightarrow{AB}|$

4. Define Scalar product of two vectors

5. If $\vec{a}$ *and* $\vec{b}$ are two adjacent sides of a parallelogram, what is its area ?

6. Evaluate: $\int \left(x^2 + \frac{3}{x}\right) dx$

7. Evaluate: $\int \frac{2x}{1+x^2}\, dx$

8. Evaluate: $\int x\, e^x dx$

PART – B

9. Prove that the equation $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ is a parabola

10. Show that the two vectors $\vec{i} - 3\vec{j} + 5\vec{k}$ and $-2\vec{i} + 6\vec{j} + 4\vec{k}$ are mutually perpendicular

11. Find the scalar triple product of the three vectors

    $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} - \vec{j} + 2\vec{k}$ and

    $\vec{c} = 3\vec{i} + \vec{j} + \vec{k}$

12. Evaluate: (i) $(\vec{i} \times \vec{j}) \bullet \vec{k}$ (ii) $(\vec{i} \times \vec{j}) \times \vec{k}$

13. Evaluate: $\int \frac{sin^2 x}{1+cosx} dx$

14. Evaluate: $\int \sin^3 x \cos x \, dx$

15. Evaluate: $\int x \log x \, dx$

16. Evaluate: $\int_1^2 (x^2 + x + 1) dx$

PART- C

17.(a) If the diameter of the circle is a line joining the points A (2, –1) and B (–4, 5),find the equation of circle. Also find the centre and radius of the circle

(b) Show that the circles $x^2 + y^2 + 2x - 4y - 3 = 0$ and $x^2 + y^2 - 8x + 6y + 7 = 0$ touch each other

(c) Show that equation $6x^2 + 13xy + 6y^2 + 8x + 7y + 2 = 0$ represents a pair of straight lines

18.(a) The position vector of the points A , B and C are

$4\vec{i} + 2\vec{j} + 3\vec{k}$, $2\vec{i} + 3\vec{j} + 4\vec{k}$ and $3\vec{i} + 4\vec{j} + 2\vec{k}$
Prove that these points form an equilateral triangle

(b) Find the projection of the vector $2\vec{i} - \vec{j} + \vec{k}$ on

$3\vec{i} - 4\vec{j} + 2\vec{k}$, Also find the angle between these two

vectors

(c) Find the work done by the force $4\vec{i} + \vec{j} - 3\vec{k}$ when it

displaces a $\vec{i} + 2\vec{j} + 3\vec{k}$ to the point $5\vec{i} + 4\vec{j} + 2\vec{k}$

19.(a) Find the unit vector perpendicular to each of the vectors $2\vec{i} + \vec{j} + \vec{k}$ *and* $\vec{i} + 2\vec{j} + \vec{k}$ .Also, find the sine of the angle between the two vectors

(b) Find the moment of the force $3\vec{i} + 4\vec{j} + 5\vec{k}$ acting through a point $\vec{i} - 2\vec{j} + 3\vec{k}$ about the point $2\vec{i} - 5\vec{j} + 3\vec{k}$

(c) If $\vec{a} = 2\vec{i} + 3\vec{j} + \vec{k}$, $\vec{b} = \vec{i} - 2\vec{j} + 3\vec{k}$ *and*

$\vec{c} = 3\vec{i} - \vec{j} + 5\vec{k}$, find $(\vec{a} \times \vec{b}) \times \vec{c}$

20. (a) Evaluate (i) $\int \frac{x^3 + 3x^2 + 2x}{x} dx$ (ii) $\int \sin^3 x \, dx$

(b) Evaluate (i) $\int \frac{(2x+1)}{x^2+x+1} dx$ (ii) $\int \frac{\sec^2(\log x)}{x} dx$

(c) Evaluate (i) $\int \frac{dx}{16 + x^2}$ (ii) $\int \frac{dx}{\sqrt{4-(x+1)^2}}$

21.(a) Evaluate (i) $\int x \sin 2x \, dx$ (ii) $\int x e^{3x} dx$

(b) Evaluate: (i) $\int x^2 \sin 4x \, dx$ (ii) $\int x^2 \cos 3x \, dx$

(c) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$

ANSWERS

PART - A

1. Find the equation of the circle with centre is (1,– 2) and radius 5 units

Answer Formula: $(x - \boldsymbol{h})^2 + (y - \boldsymbol{k})^2 = \mathbf{r^2}$ **-----------(1 mark)**

Here $(h, k) = (1,-2)$ and r = 5

$\therefore (x - \mathbf{1})^2 + (y + \mathbf{2})^2 = \mathbf{5}^2$

$(x - \mathbf{1})(x - \mathbf{1}) + (y + \mathbf{2})(y + \mathbf{2}) = \mathbf{25}$

$(x^2 - \boldsymbol{x} - \boldsymbol{x} + 1) + (y^2 + \mathbf{2}\boldsymbol{y} + \mathbf{2}\boldsymbol{y} + 4) = 25$

$x^2 - \mathbf{2}\boldsymbol{x} + 1 + y^2 + 4y + 4 - 25 = 0$

$x^2 + y^2 - 2x + 4y - 20 = 0$ **-----------(1 mark)**

2. Show that the circles $x^2 + y^2 - 4x + 2y + 5 = 0$ and $x^2 + y^2 - 4x + 2y - 8 = 0$ are concentric circles

Answer Given $x^2 + y^2 - 4x + 2y + \mathbf{5} = 0$

$x^2 + y^2 - 4x + 2y - \mathbf{8} = 0$

Here $c_1 = 5$ and $c_2 = -8$ **-----------(1 mark)**

$\because c_1 \neq c_2$

$\therefore$ Given circles are concentric **-----------(1 mark)**

3. If $2\vec{i} - \vec{j} + 3\vec{k}$ and $5\vec{i} + \vec{j} - 2\vec{k}$ *are* the position vectors of the points A and B , find $\overrightarrow{AB}$ and its modulus $|\overrightarrow{AB}|$

Answer Let $\overrightarrow{OA} = 2\vec{i} - \vec{j} + 3\vec{k}$ and $\overrightarrow{OB} = 5\vec{i} + \vec{j} - 2\vec{k}$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$\overrightarrow{AB} = (5\vec{i} + \vec{j} - 2\vec{k}) - (2\vec{i} - \vec{j} + 3\vec{k})$$

$$\overrightarrow{AB} = 5\vec{i} + \vec{j} - 2\vec{k} - 2\vec{i} + \vec{j} - 3\vec{k}$$

$$\overrightarrow{AB} = 3\vec{i} + 2\vec{j} - 5\vec{k}$$ **-----------(1 mark)**

$$|\overrightarrow{AB}| = \sqrt{(3)^2 + (2)^2 + (-5)^2}$$

$$|\overrightarrow{AB}| = \sqrt{9 + 4 + 25} = \sqrt{38} \text{ units}$$ **-----------(1 mark)**

4. Define Scalar product of two vectors

Answer The Scalar product of two vectors $\vec{a}$ and $\vec{b}$ is defined as $\vec{a} \bullet \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, where θ is the angle between the vectors **-----------(2 marks)**

5. If $\vec{a}$ *and* $\vec{b}$ are two adjacent sides of a parallelogram, what is its area ?

Answer Area of a parallelogram is $|\vec{a} \times \vec{b}|$ **------(2 marks)**

6. Evaluate: $\int \left(x^2 + \frac{3}{x}\right) dx$

Answer $\int \left(x^2 + \frac{3}{x}\right) dx = \frac{x^3}{3} + 3\log x + c$ **-------(2 marks)**

7. Evaluate: $\int \frac{2x}{1+x^2}\, dx$

Answer $\int \frac{2x}{1+x^2}\, dx$ ----- (1)

Put $t = 1 + x^2$ $\frac{dt}{dx} = 2x$ $dt = 2x\ dx$	$(1) \Rightarrow \int \frac{1}{t}\, dt$ $= \log(t) + c$ -----------**(1 mark)** $= \log(1 + x^2) + c$-----------**(1 mark)**

8.Evaluate: $\int x\, e^x dx$

Answer

Let $u = x$ $\frac{du}{dx} = 1$ $du = dx$	$dv = e^x\, dx$ $\int dv = \int e^x\, dx$ $v = e^x$

-----------**(1 mark)**

Formula: $\int \boldsymbol{u\, dv} = \boldsymbol{uv} - \int \boldsymbol{v\, du}$

$\int x\, e^x\, dx = x\, e^x - \int e^x\, dx$

$= x\, e^x - e^x + c$-----------**(1 mark)**

PART - B

9. Prove that the equation $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ is a parabola

Answer Given $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ —— (1)

We know that, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ —— (2)

On comparing (1) & (2) ,we get

a = 1, b = 9, 2h = 6 ⇒ h = 3 -----------**(1 mark)**

Condition for a parabola is $\mathbf{h^2 - ab = 0}$

$$h^2 - ab = (3)^2 - (1)(9)$$

$$= 0 \quad \text{----------(2 marks)}$$

$\therefore$ The given equation represents a parabola

10. Show that the two vectors $\vec{i} - 3\vec{j} + 5\vec{k}$ and $-2\vec{i} + 6\vec{j} + 4\vec{k}$ are mutually perpendicular.

Answer

Let $\vec{a} = \vec{i} - 3\vec{j} + 5\vec{k}$, $\vec{b} = -2\vec{i} + 6\vec{j} + 4\vec{k}$

$\vec{a} \bullet \vec{b} = (\vec{i} - 3\vec{j} + 5\vec{k}) \bullet (-2\vec{i} + 6\vec{j} + 4\vec{k})$----(**1 mark)**

$= (1)(-2) + (-3)(6) + (5)(4)$

$= -2 - 18 + 20$

$\vec{a} \bullet \vec{b} = 0 \Rightarrow \vec{a}$ *and* $\vec{b}$ are perpendicular ------(**2 marks)**

11. Find the scalar triple product of the three vectors $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} - \vec{j} + 2\vec{k}$ *and* $\vec{c} = 3\vec{i} + \vec{j} + \vec{k}$

Answer

Let $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} - \vec{j} + 2\vec{k}$,

$\vec{c} = 3\vec{i} + \vec{j} + \vec{k}$

$$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} 2 & -1 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} \quad \text{-----------(1 mark)}$$

$$= 2\begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix}$$

$$= 2(-1 - 2) + 1(1 - 6) + 1(1 + 3)$$

$$= 2(-3) + 1(-5) + 1(4)$$

$$[\vec{a}, \vec{b}, \vec{c}] = -6 - 5 + 4 = -7 \text{----------(2 marks)}$$

12. Evaluate: (i) $(\vec{i} \times \vec{j}) \bullet \vec{k}$ (ii) $(\vec{i} \times \vec{j}) \times \vec{k}$

Answer

(i) $(\vec{i} \times \vec{j}) \bullet \vec{k}$

$= \vec{k} \bullet \vec{k} = 1$ -----------($1\frac{1}{2}$ marks)

(ii) $(\vec{i} \times \vec{j}) \times \vec{k}$

$= \vec{k} \times \vec{k} = 0$ -----------($1\frac{1}{2}$ marks)

13.Evaluate: $\int \frac{sin^2x}{1+cosx} dx$

Answer

$\int \frac{sin^2x}{1+cosx} dx = \int \frac{1-cos^2x}{1+cosx} dx$ -----------(1 mark)

$= \int \frac{(1+cosx)(1-cosx)}{1+cosx} dx$-----------(1 mark)

$= \int (1 - cosx)dx$

$= x - sinx + c$ -----------(1 mark)

14. Evaluate: $\int \sin^3 x \cos x \, dx$

Answer $\int \sin^3 x \cos x \, dx$ ----- (1)

Put $t = \sin x$ $\frac{dt}{dx} = \cos x$ $dt = \cos x \, dx$---(1 mark)	$(1) \Rightarrow \int t^3 \, dt$ $= \frac{t^4}{4} + c$ ------(1 mark) $= \frac{(sinx)^4}{4} + c$ ----(1 mark)

15. Evaluate: $\int x \log x \, dx$

Answer

Let $u = \log x$ $\frac{du}{dx} = \frac{1}{x}$ $du = \frac{dx}{x}$	$\int dv = \int x\,dx$ $v = \frac{x^2}{2}$

-----------(**1 mark)**

Formula: $\int \boldsymbol{u\,dv} = \boldsymbol{uv} - \int \boldsymbol{v\,du}$ -----------(**1 mark)**

$$\int x \log x \; dx = \log x \left(\frac{x^2}{2}\right) - \int \frac{x^2}{2}\,\frac{dx}{x}$$

$$= \frac{x^2}{2}\log x - \frac{1}{2}\int x\,dx$$

$$= \frac{x^2}{2}\log x - \frac{1}{2}\left(\frac{x^2}{2}\right) + c$$

$$= \frac{x^2}{2}\log x - \frac{x^2}{4} + c$$ -----------(**1 mark)**

16. Evaluate: $\int_1^2 (x^2 + x + 1)dx$

Answer

$$\int_1^2 (x^2 + x + 1)dx$$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2} + x\right]_1^2$$ -----------(**1 mark)**

$$= \left[\left(\frac{2^3}{3} + \frac{2^2}{2} + 2\right) - \left(\frac{1^3}{3} + \frac{1^2}{2} + 1\right)\right]$$ -------(**1 mark)**

$$= \left[\left(\frac{8}{3} + 2 + 2\right) - \left(\frac{1}{3} + \frac{1}{2} + 1\right)\right]$$

$$= \left(\frac{8+6+6}{3}\right) - \left(\frac{2+3+6}{6}\right)$$

$$= \left(\frac{20}{3}\right) - \left(\frac{11}{6}\right)$$

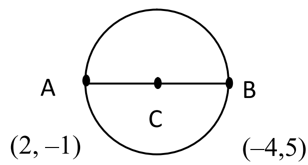
$$= \frac{40-11}{6}$$

$$= \frac{29}{6}$$ -----------(**1 mark)**

PART - C

17.(a). If the diameter of the circle is a line joining the points A (2, –1) and B (–4, 5),find the equation of circle. Also find the centre and radius of the circle.

Answer



Formula: $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ ----------**(1 mark)**

Here $(x_1, y_1) = (2, -1)$ and $(x_2, y_2) = (-4, 5)$

$$(x - \mathbf{2})(x + \mathbf{4}) + (y + \mathbf{1})(y - \mathbf{5}) = 0$$

$$(x^2 + 4x - 2x - 8) + (y^2 - 5y + y - 5) = 0$$

$$x^2 + 2x - 8 + y^2 - 4y - 5 = 0$$

$$x^2 + y^2 + 2x - 4y - 13 = 0 \text{ — (1)}$$ ---- **(2 marks)**

which is the required equation of the circle

To find the centre and radius:

$$(1) \Rightarrow x^2 + y^2 + 2x - 4y - 13 = 0$$

We know that $x^2 + y^2 + 2gx + 2fy + c = 0$ — (2)

On comparing (1) and (2) , we get

$2g = 2$	$2f = -4$	$c = -13$
$g = \frac{2}{2} = 1$	$f = \frac{-4}{2} = -2$	

$\therefore$ Centre C = $(-g, -f)$ = $(-1, 2)$ ----------(**1 mark)**

$$Radius\ \ r = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{(1)^2 + (-2)^2 - (-13)} = \sqrt{1 + 4 + 13} = \sqrt{18} \text{ units}$$ **(1 mark)**

17.(b). Show that the circles $x^2 + y^2 + 2x - 4y - 3 = 0$ and $x^2 + y^2 - 8x + 6y + 7 = 0$ touch each other

Answer

Circle 1	Circle 2
$x^2 + y^2 + 2x - 4y - 3 = 0$ Centre $= \left(\frac{-coeff\ of\ x}{2}, \frac{-coeff\ of\ y}{2}\right)$ $= \left(\frac{-2}{2}, \frac{-(-4)}{2}\right)$ C_1 = $(-1,2)$ --------(**1 mark)**	$x^2 + y^2 - 8x + 6y + 7 = 0$ Centre $= \left(\frac{-coeff\ of\ x}{2}, \frac{-coeff\ of\ y}{2}\right)$ $= \left(\frac{-(-8)}{2}, \frac{6}{2}\right)$ C_2 = $(4, -3)$ ---------(**1 mark)**
Radius r = $\sqrt{g^2 + f^2 - c}$ Here g = 1, f = –2 , c = – 3 $r_1 = \sqrt{(1)^2 + (-2)^2 - (-3)}$ $= \sqrt{1 + 4 + 3}$ $= \sqrt{8} = 2\sqrt{2}$ **units** **-------- (1 mark)**	Radius r = $\sqrt{g^2 + f^2 - c}$ Here g = – 4, f = 3 , c = 7 $r_2 = \sqrt{(-4)^2 + (3)^2 - (7)}$ $= \sqrt{16 + 9 - 7}$ $= \sqrt{18} = 3\sqrt{2}$ **units** **--------- (1 mark)**

Distance between the centres $C_1 C_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here $C_1 = (-1,2) = (x_1, y_1)$, $C_2 = (4, -3) = (x_2, y_2)$

$$C_1 C_2 = \sqrt{(4+1)^2 + (-3-2)^2}$$

$$= \sqrt{(5)^2 + (-5)^2}$$

$$= \sqrt{25+25}$$

$$= \sqrt{50} = 5\sqrt{2} \text{ units}$$

$\therefore C_1 C_2 = r_1 + r_2 = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$ ----------**(1 mark)**

$\therefore$ The circles touch each other externally

17.(c). Show that equation $6x^2 + 13xy + 6y^2 + 8x + 7y + 2 = 0$ represents a pair of straight lines

Answer

Given $6x^2 + 13xy + 6y^2 + 8x + 7y + 2 = 0$ —— (1)

We know that, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ——(2)

On comparing(1) & (2) ,we get $\mathbf{2h} = 13$; $\mathbf{2g} = 8$; $\mathbf{2f} = 7$

a = 6, $\Rightarrow$ **2a** = 2(6) = **12**

b = 6, $\Rightarrow$ **2b** = 2(6) = **12**

c = 2, $\Rightarrow$ **2c** = 2(2) = **4** ----------**(1 mark)**

The condition for pair of straight line is

$$\therefore \begin{vmatrix} \mathbf{2a} & \mathbf{2h} & \mathbf{2g} \\ \mathbf{2h} & \mathbf{2b} & \mathbf{2f} \\ \mathbf{2g} & \mathbf{2f} & \mathbf{2c} \end{vmatrix} = \mathbf{0}$$ ----------**(1 mark)**

$$\begin{vmatrix} 12 & 13 & 8 \\ 13 & 12 & 7 \\ 8 & 7 & 4 \end{vmatrix} = 0$$ **----------(1 mark)**

$$12\begin{vmatrix} 12 & 7 \\ 7 & 4 \end{vmatrix} - 13\begin{vmatrix} 13 & 7 \\ 8 & 4 \end{vmatrix} + 8\begin{vmatrix} 13 & 12 \\ 8 & 7 \end{vmatrix} = 0$$

$$12\,[48 - 49] - 13\,[52 - 56] + 8\,[91 - 96] = 0$$

$$12\,[-1] - 13\,[-4] + 8\,[-5] = 0$$

$$-12 + 52 - 40 = 0$$

$$0 = 0$$ **---------(2 marks)**

⇐ The given equation represents a **pair of straight lines**

18.(a). The position vector of the points A , B and C are

$4\vec{i} + 2\vec{j} + 3\vec{k}$, $2\vec{i} + 3\vec{j} + 4\vec{k}$ and $3\vec{i} + 4\vec{j} + 2\vec{k}$
Prove that these points form an equilateral triangle

Answer Let $\overrightarrow{OA} = 4\vec{i} + 2\vec{j} + 3\vec{k}$, $\overrightarrow{OB} = 2\vec{i} + 3\vec{j} + 4\vec{k}$

$\overrightarrow{OC} = 3\vec{i} + 4\vec{j} + 2\vec{k}$ **----------(1 mark)**

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$\overrightarrow{AB} = (2\vec{i} + 3\vec{j} + 4\vec{k}) - (4\vec{i} + 2\vec{j} + 3\vec{k})$$

$$\overrightarrow{AB} = 2\vec{i} + 3\vec{j} + 4\vec{k} - 4\vec{i} - 2\vec{j} - 3\vec{k}$$

$$\overrightarrow{AB} = -2\vec{i} + \vec{j} + \vec{k}$$

$$|\overrightarrow{AB}| = \sqrt{(-2)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{4 + 1 + 1} = \sqrt{6}$$ **----------(1 mark)**

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$\overrightarrow{BC} = (3\vec{i} + 4\vec{j} + 2\vec{k}) - (2\vec{i} + 3\vec{j} + 4\vec{k})$

$\overrightarrow{BC} = 3\vec{i} + 4\vec{j} + 2\vec{k} - 2\vec{i} - 3\vec{j} - 4\vec{k}$

$\overrightarrow{BC} = \vec{i} + \vec{j} - 2\vec{k}$

$|\overrightarrow{BC}| = \sqrt{(1)^2 + (1)^2 + (-2)^2}$

$= \sqrt{1 + 1 + 4} = \sqrt{6}$ **----------(1 mark)**

$\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC}$

$\overrightarrow{CA} = (4\vec{i} + 2\vec{j} + 3\vec{k}) - (3\vec{i} + 4\vec{j} + 2\vec{k})$

$\overrightarrow{CA} = 4\vec{i} + 2\vec{j} + 3\vec{k} - 3\vec{i} - 4\vec{j} - 2\vec{k}$

$\overrightarrow{CA} = \vec{i} - 2\vec{j} + \vec{k}$

$|\overrightarrow{CA}| = \sqrt{(1)^2 + (-2)^2 + (1)^2}$

$= \sqrt{1 + 4 + 1} = \sqrt{6}$ **----------(1 mark)**

$\therefore |\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CA}| = \sqrt{6}$ **----------(1 mark)**

$\therefore$ A B C is an equilateral triangle

18.(b). Find the projection of the vector $2\vec{i} - \vec{j} + \vec{k}$ on $3\vec{i} - 4\vec{j} + 2\vec{k}$, Also find the angle between these two vectors

Answer

Formula: projection of $\vec{a}$ on $\vec{b}$ = $\dfrac{\vec{a} \bullet \vec{b}}{|\vec{b}|}$ **--(1 mark)**

Let $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = 3\vec{i} - 4\vec{j} + 2\vec{k}$

$$\vec{a} \bullet \vec{b} = (2\vec{\imath} - \vec{\jmath} + \vec{k}) \bullet (3\vec{\imath} - 4\vec{\jmath} + 2\vec{k})$$

$$= (2)(3) + (-1)(-4) + (1)(2)$$

$$= 6 + 4 + 2$$

$$\boldsymbol{\vec{a} \bullet \vec{b} = 12}$$

$$|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{4 + 1 + 1} = \boldsymbol{\sqrt{6}}$$

$$|\vec{b}| = \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{(3)^2 + (-4)^2 + (2)^2} = \sqrt{9 + 16 + 4} = \boldsymbol{\sqrt{29}}$$

-----(1 mark)

$\therefore$ Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a} \bullet \vec{b}}{|\vec{b}|} = \frac{12}{\sqrt{29}}$ units **-----(1 mark)**

Angle between $\vec{a}$ and $\vec{b}$ is $\cos\theta = \frac{\vec{a} \bullet \vec{b}}{|\vec{a}||\vec{b}|}$ **-----(1 mark)**

$$\cos\theta = \frac{\vec{a} \bullet \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\cos\theta = \frac{12}{\sqrt{6}\sqrt{29}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{12}{\sqrt{6}\sqrt{29}}\right)$$ **-----(1 mark)**

18.(c) Find the work done by the force $4\vec{\imath} + \vec{\jmath} - 3\vec{k}$ when it displaces a $\vec{\imath} + 2\vec{\jmath} + 3\vec{k}$ to the point $5\vec{\imath} + 4\vec{\jmath} + 2\vec{k}$

Answer Formula: Work done W = $\vec{F} \bullet \vec{d}$ **----------(1 mark)**

$\vec{F}$ (force)	$\vec{F} = 4\vec{i} + \vec{j} - 3\vec{k}$

$\vec{d}$ = To the point – From the point

$= (5\vec{i} + 4\vec{j} + 2\vec{k}) - (\vec{i} + 2\vec{j} + 3\vec{k})$

$= 5\vec{i} + 4\vec{j} + 2\vec{k} - \vec{i} - 2\vec{j} - 3\vec{k}$

$= 4\vec{i} + 2\vec{j} - \vec{k}$ **----------(2 marks)**

Work done W = $\vec{F} \bullet \vec{d}$

$= (4\vec{i} + \vec{j} - 3\vec{k}) \bullet (4\vec{i} + 2\vec{j} - \vec{k})$

$= (4)(4) + (1)(2) + (-3)(-1)$

$= 16 + 2 + 3$

Work done W = 21units **----------(2 marks)**

19.(a). Find the unit vector perpendicular to each of the vectors $2\vec{i} + \vec{j} + \vec{k}$ *and* $\vec{i} + 2\vec{j} + \vec{k}$ .Also, find the sine of the angle between the two vectors

Answer Let $\vec{a} = 2\vec{i} + \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} + \vec{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$= \vec{i}(1 - 2) - \vec{j}(2 - 1) + \vec{k}(4 - 1)$

$= \vec{i}(-1) - \vec{j}(1) + \vec{k}(3)$

$\vec{a} \times \vec{b} = -\vec{i} - \vec{j} + 3\vec{k}$ **----------(1 mark)**

$|\vec{a} \times \vec{b}| = \sqrt{x^2 + y^2 + z^2}$ $\{x = -1, y = -1, z = 3\}$

$$= \sqrt{(-1)^2 + (-1)^2 + (3)^2}$$

$$= \sqrt{1 + 1 + 9}$$

$$|\vec{a} \times \vec{b}| = \sqrt{11}$$ **----------(1 mark)**

$$|\vec{a}| = \sqrt{x^2 + y^2 + z^2} \qquad \{x = 2, y = 1, z = 1\}$$

$$= \sqrt{(2)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{4 + 1 + 1}$$

$$|\vec{a}| = \sqrt{6}$$

$$|\vec{b}| = \sqrt{x^2 + y^2 + z^2} \qquad \{x = 1, y = 2, z = 1\}$$

$$= \sqrt{(1)^2 + (2)^2 + (1)^2}$$

$$= \sqrt{1 + 4 + 1}$$

$$|\vec{b}| = \sqrt{6}$$ **----------(1 mark)**

∴The unit vector perpendicular to both $\vec{a}$ *and* $\vec{b}$ *is*

$$\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{-\vec{i} - \vec{j} + 3\vec{k}}{\sqrt{11}}$$ **----------(1 mark)**

∴ Sine of the angle between $\vec{a}$ *and* $\vec{b}$ *is* $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}|\,|\vec{b}|}$

$$\sin\theta = \frac{\sqrt{11}}{\sqrt{6}\,\sqrt{6}}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{\sqrt{11}}{\sqrt{6}\,\sqrt{6}}\right)$$ **----------(1 mark)**

19.(b). Find the moment of the force $3\vec{i} + 4\vec{j} + 5\vec{k}$ acting through a point $\vec{i} - 2\vec{j} + 3\vec{k}$ about the point $2\vec{i} - 5\vec{j} + 3\vec{k}$

Answer Moment of the force = $\vec{r} \times \vec{F}$ **-----(1 mark)**

Given: Acting point = $\vec{i} - 2\vec{j} + 3\vec{k}$

About point = $2\vec{i} - 5\vec{j} + 3\vec{k}$

$\vec{F}$ **(force)**	$\vec{F} = 3\vec{i} + 4\vec{j} + 5\vec{k}$
$\vec{r}$ = ***Acting point – About point*** $= (\vec{i} - 2\vec{j} + 3\vec{k}) - (2\vec{i} - 5\vec{j} + 3\vec{k})$ $= \vec{i} - 2\vec{j} + 3\vec{k} - 2\vec{i} + 5\vec{j} - 3\vec{k}$ $= -\vec{i} + 3\vec{j} + 0\vec{k}$ **----------(1 mark)**	

$$\vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 3 & 0 \\ 3 & 4 & 5 \end{vmatrix}$$ **---------(1 mark)**

$$= \vec{i}\begin{vmatrix} 3 & 0 \\ 4 & 5 \end{vmatrix} - \vec{j}\begin{vmatrix} -1 & 0 \\ 3 & 5 \end{vmatrix} + \vec{k}\begin{vmatrix} -1 & 3 \\ 3 & 4 \end{vmatrix}$$

$$= \vec{i}(15 - 0) - \vec{j}(-5 - 0) + \vec{k}(-4 - 9)$$

$$= \vec{i}(15) - \vec{j}(-5) + \vec{k}(-13)$$

$$\vec{r} \times \vec{F} = 15\vec{i} + 5\vec{j} - 13\vec{k}$$ **----------(2 marks)**

19.(c). If $\vec{a} = 2\vec{i} + 3\vec{j} + \vec{k}$, $\vec{b} = \vec{i} - 2\vec{j} + 3\vec{k}$ *and* $\vec{c} = 3\vec{i} - \vec{j} + 5\vec{k}$, find $(\vec{a} \times \vec{b}) \times \vec{c}$

Answer

Given $\vec{a} = 2\vec{i} + 3\vec{j} + \vec{k}$, $\vec{b} = \vec{i} - 2\vec{j} + 3\vec{k}$, $\vec{c} = 3\vec{i} - \vec{j} + 5\vec{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 1 \\ 1 & -2 & 3 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} 3 & 1 \\ -2 & 3 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix}$$

$$= \vec{i}(9+2) - \vec{j}(6-1) + \vec{k}(-4-3)$$

$$= \vec{i}(11) - \vec{j}(5) + \vec{k}(-7)$$

$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = 11\vec{i} - 5\vec{j} - 7\vec{k}$$ **--------($2\frac{1}{2}$ marks)**

$$(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 11 & -5 & -7 \\ 3 & -1 & 5 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} -5 & -7 \\ -1 & 5 \end{vmatrix} - \vec{j}\begin{vmatrix} 11 & -7 \\ 3 & 5 \end{vmatrix} + \vec{k}\begin{vmatrix} 11 & -5 \\ 3 & -1 \end{vmatrix}$$

$$= \vec{i}(-25-7) - \vec{j}(55+21) + \vec{k}(-11+15)$$

$$= \vec{i}(-32) - \vec{j}(76) + \vec{k}(4)$$

$$\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = -32\vec{i} - 76\vec{j} + 4\vec{k}$$ **--------($2\frac{1}{2}$ marks)**

20.(a). Evaluate (i) $\int \frac{x^3+3x^2+2x}{x} dx$ (ii) $\int \sin^3 x \, dx$

Answer

(i) $\int \frac{x^3+3x^2+2x}{x} dx$

$$= \int \frac{x^3}{x} + \frac{3x^2}{x} + \frac{2x}{x} dx$$

$$= \int (x^2 + 3x + 2) dx$$ **--------(1 mark)**

$$= \frac{x^3}{3} + 3\frac{x^2}{2} + 2x + c$$ **-------($1\frac{1}{2}$ marks)**

Answer

(ii) $\int \sin^3 x \, dx$

Formula: $\sin^3 A = \frac{3\sin A - \sin 3A}{4}$

Here $A = x$ $\sin^3 x = \frac{3\sin x - \sin 3x}{4}$

$$\int \sin^3 x \, dx = \int \frac{3\sin x - \sin 3x}{4} dx$$ **--------(1 mark)**

$$= \frac{1}{4}\int (3sinx - sin3x)\ dx$$

$$= \frac{1}{4}\left[-3cosx + \frac{cos3x}{3}\right] + c$$ ------($1\frac{1}{2}$ marks)

20.(b). Evaluate (i) $\int \frac{(2x+1)}{x^2+x+1}\,dx$ (ii) $\int \frac{sec^2(logx)}{x}\,dx$

Answer (i) $\int \frac{(2x+1)}{x^2+x+1}\,dx$ ----- (1)

Put $t = x^2 + x + 1$ $\frac{dt}{dx} = 2x + 1$ $dt = (2x + 1)dx$ -------(**1 mark**)	$(1) \Rightarrow \int \left(\frac{1}{t}\right) dt$ $= \log(t) + c$ $= \log(x^2 + x + 1) + c$ -------($1\frac{1}{2}$ **marks**)

Answer (ii) $\int \frac{sec^2(logx)}{x}\,dx$ ----- (1)

Put $t = logx$ $\frac{dt}{dx} = \frac{1}{x}$ $dt = \frac{1}{x}\,dx$ --------(**1 mark**)	$(1) \Rightarrow \int sec^2t\,dt$ $= \tan(t) + c$ $= \tan(logx) + c$ -------($1\frac{1}{2}$ **marks**)

20.(c).Evaluate (i) $\int \frac{dx}{16 + x^2}$ (ii) $\int \frac{dx}{\sqrt{4-(x+1)^2}}$

Answer (i) $\int \frac{dx}{16 + x^2}$

$$Formula: \int \frac{dx}{a^2 + x^2} = \frac{1}{a} tan^{-1}\left(\frac{x}{a}\right) + c$$

Where $x = x$ and a = 4 --------($1\frac{1}{2}$ marks)

$$\int \frac{dx}{16 + x^2} = \int \frac{dx}{4^2 + x^2} = \frac{1}{4} tan^{-1}\left(\frac{x}{4}\right) + c$$ --------(1 mark)

Answer (ii) $\int \frac{dx}{\sqrt{4-(x+1)^2}} = \int \frac{dx}{\sqrt{(2)^2-(x+1)^2}}$ ------ (1)

Put $t = x + 1$ $\frac{dt}{dx} = 1$ $dt = 1\,dx$ --------(1 mark)	$(1) \Rightarrow \int \frac{dt}{\sqrt{(2)^2-(t)^2}}$ $= sin^{-1}\left(\frac{t}{2}\right) + c$ $= sin^{-1}\left(\frac{x+1}{2}\right) + c$ -------($1\frac{1}{2}$ marks)

21.(a). Evaluate (i) $\int x \sin 2x\, dx$ **(ii)** $\int x\, e^{3x} dx$

Answer **(i)** $\int x \sin 2x\, dx$

Let $u = x$

$\frac{du}{dx} = 1$

$du = dx$

$\int dv = \int \sin 2x\, dx$

$v = \frac{-\cos 2x}{2}$

--------($1\frac{1}{2}$ marks)

Formula: $\int u\, dv = uv - \int v\, du$

$$\int x \sin 2x\, dx = x\left(\frac{-\cos 2x}{2}\right) - \int \frac{-\cos 2x}{2} dx$$

$$= \frac{-x\cos 2x}{2} + \frac{1}{2}\int \cos 2x\, dx$$

$$= \frac{-x\cos 2x}{2} + \frac{1}{2}\left(\frac{\sin 2x}{2}\right) + c$$

$$= \frac{-x\cos 2x}{2} + \frac{\sin 2x}{4} + c$$ **------(1 mark)**

Answer

(ii) $\int x\, e^{3x} dx$

Let $u = x$

$\frac{du}{dx} = 1$

$du = dx$

$\int dv = \int e^{3x} dx$

$v = \frac{e^{3x}}{3}$

--------($1\frac{1}{2}$ marks)

Formula: $\int u\, dv = uv - \int v\, du$

$$\int x\, e^{3x}\, dx = \frac{x\, e^{3x}}{3} - \int \frac{e^{3x}}{3}\, dx$$

$$= \frac{x\, e^{3x}}{3} - \frac{1}{3}\int e^{3x}\, dx$$

$$= \frac{x\, e^{3x}}{3} - \frac{1}{3}\left(\frac{e^{3x}}{3}\right) + c$$

$$= \frac{x\, e^{3x}}{3} - \frac{e^{3x}}{9} + c$$ **------(1 mark)**

21.(b). Evaluate: (*i*) $\int x^2 \sin 4x\, dx$ (ii) $\int x^2 \cos 3x\, dx$

Answer

(i) $\int x^2 \sin 4x\, dx$

Let $u = x^2$

$u' = 2x$

$u'' = 2$

$\int dv = \int \sin 4x\, dx$

$v = \frac{-\cos 4x}{4}$

$v_1 = \frac{-\sin 4x}{16}$

$v_2 = \frac{\cos 4x}{64}$ **----($1\frac{1}{2}$ marks)**

Formula: $\int u\,dv = uv - u'\,v_1 + u''v_2 - \cdots$

$$\int x^2 \sin 4x\, dx = x^2\left(\frac{-\cos 4x}{4}\right) - 2x\left(\frac{-\sin 4x}{16}\right) + \frac{2\cos 4x}{64} + c$$

$$= \frac{-x^2\cos 4x}{4} + \frac{x\sin 4x}{8} + \frac{\cos 4x}{32} + c \text{--------(1 mark)}$$

Answer **(i)** $\int x^2 \cos 3x\ dx$

Let $u = x^2$

$u' = 2x$

$u'' = 2$

$\int dv = \int \cos 3x\ dx$

$v = \frac{\sin 3x}{3}$

$v_1 = \frac{-\cos 3x}{9}$

$v_2 = \frac{-\sin 3x}{27}$

--------($1\frac{1}{2}$ marks)

Formula: $\int u\,dv = uv - u'\,v_1 + u''v_2 - \cdots$

$$\int x^2 \cos 3x\, dx = \frac{x^2 \sin 3x}{3} - 2x\left(\frac{-\cos 3x}{9}\right) + 2\left(\frac{-\sin 3x}{27}\right) + c$$

$$= \frac{x^2\sin 3x}{3} + \frac{2x\cos 3x}{9} - \frac{2\sin 3x}{27} + c \text{ --------(1 mark)}$$

21.(c). Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}\, dx$

Answer

Let $\mathbf{I} = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}\, dx$ - - (1) **--------(1 mark)**

By property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ -----------**(1 mark)**

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x+\sin x}dx \text{ - - - - - (2)}$$ -----------**(1 mark)**

Adding (1) and (2),we get

$$I + I = \int_0^{\frac{\pi}{2}} \frac{\sin x dx}{\sin x+\cos x} + \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x}dx$$

$$2I = \int_0^{\frac{\pi}{2}} \left(\frac{\sin x+\cos x}{\sin x+\cos x}\right)dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$ -----------**(1 mark)**

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$2I = \frac{\pi}{2} - 0$$

$$2I = \frac{\pi}{2} \quad \Rightarrow I = \frac{\pi}{4}$$ -----------**(1 mark)**