OCTOBER 2019

$$\frac{Time - Three\ hours}{(Maximum\ Marks: 75)}$$

[N.B:- (1) Answer any FIVE questions in each of PART-A& PART-B and any two divisions of each questions in PART-C

(2) Each questions carries 2(two)marks in PART-A ,3(three)marks in PART-B and 5(five)marks for each division in PART-C.]

PART – A

1. Find the centre and radius of the circle $x^2 + y^2 + 4x + 4y - 1 = 0$
2. Show that the circles $x^2 + y^2 - 2x + 4y - 3 = 0$ and $x^2 + y^2 - 2x + 4y + 5 = 0$ are concentric circles
3. If $\vec{a} = 3\vec{i} + 2\vec{j} + \vec{k}$ and $\vec{b} = \vec{i} + 3\vec{j} + \vec{k}$, find $3\vec{a} + \vec{b}$
4. What are the values of $\vec{i} \bullet \vec{j}$ and $\vec{k} \bullet \vec{k}$?
5. If $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 10$, find the angle between $\vec{a}$ and $\vec{b}$
6. Evaluate: $\int (x^2 + cosx)\, dx$
7. Evaluate: $\int \frac{1}{\sqrt{x^2+9}} dx$

8 Evaluate: $\int_1^2 (x + x^2) dx$

PART – B

9. Prove that the equation $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ is a parabola
10. Show that the two vectors $\vec{i} - 3\vec{j} + 5\vec{k}$ and $-2\vec{i} + 6\vec{j} + 4\vec{k}$ are mutually perpendicular.
11. Find the area of the triangle whose adjacent sides are $2\vec{i} + 3\vec{j} - \vec{k}$ and $\vec{i} + 3\vec{j} + \vec{k}$

12. Find the value of 'm' if the vectors $\vec{i} + 2\vec{j} - \vec{k}$, $2\vec{i} + m\vec{j} - 3\vec{k}$ and $3\vec{i} + \vec{j} + 4\vec{k}$ are coplanar

13. Evaluate $\int_0^{\frac{\pi}{2}} \frac{cos^2 x}{1+sinx}\, dx$

14. Evaluate $\int \frac{e^x}{1+e^x} dx$

15. Evaluate: $\int x \log x\, dx$

16. Evaluate: $\int (x^2 + x + 1)(x + 5)\, dx$

PART- C

17.(a) Find the equation of the circle passing through the point A (2, 3) and having its centre at C (4, 1)

(b) Prove that the circles $x^2 + y^2 - 8x + 6y - 23 = 0$ and $x^2 + y^2 - 2x - 5y + 16 = 0$ cut orthogonally

(c) Show that the equation $3x^2 + 7xy + 2y^2 + 5x + 5y + 2 = 0$ represents a pair of straight lines

18.(a) The position vector of the points A , B and C are $4\vec{i} + 2\vec{j} + 3\vec{k}$, $2\vec{i} + 3\vec{j} + 4\vec{k}$ and $3\vec{i} + 4\vec{j} + 2\vec{k}$. Prove that these points form an equilateral triangle

(b) If $\vec{a} = 2\vec{i} + \vec{j} + 3\vec{k}$, and $\vec{b} = \vec{i} - 4\vec{j} - 6\vec{k}$, find the projection of $\vec{a}$ on $\vec{b}$ .Also find the angle between the two vectors $\vec{a}$ and $\vec{b}$.

(c) Find the work done by the force $\vec{i} + 3\vec{j} - \vec{k}$ when it displaces a particle from the point $2\vec{i} - 6\vec{j} + 7\vec{k}$ to the point $3\vec{i} - \vec{j} - 5\vec{k}$

19.(a) Find the unit vector perpendicular to each of the vectors $3\vec{i} + 3\vec{j} + \vec{k}$ and $2\vec{i} - 5\vec{j} + 3\vec{k}$.

(b) Find the moment of the force $6\vec{i} + \vec{j} + \vec{k}$ acting through a point $\vec{i} + 2\vec{j} + 3\vec{k}$ about the point $-\vec{i} - \vec{j} + \vec{k}$

(c) If $\vec{a} = 2\vec{i} - \vec{j} + 3\vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} - \vec{k}$, $\vec{c} = 3\vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{d} = \vec{i} + 3\vec{j} + 4\vec{k}$ find $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$

20. (a) Evaluate (i) $\int (x-1)(2x+3)\, dx$ (ii) $\int 2 \sin 3x \cos x \, dx$

(b) Evaluate: (i) $\int \frac{6x^2-1}{2x^3-x+5} dx$ (ii) $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

(c) Evaluate: (i) $\int \frac{dx}{16+x^2}$ (ii) $\int \frac{dx}{\sqrt{36-x^2}}$

21.(a) Evaluate: (i) $\int x\, e^{4x}\, dx$ (ii) $\int x \sin 2x \, dx$

(b) Evaluate: (i) $\int x^2 e^x dx$ (ii) $\int x^2 \cos 3x \, dx$

(c) Evaluate: $\int_0^{\frac{\pi}{4}} \tan x \sec^2 x \, dx$

ANSWERS

PART - A

1. Find the centre and radius of the circle $x^2 + y^2 + 4x + 4y - 1 = 0$

Answer Given $x^2 + y^2 + 4x + 4y - 1 = 0$ ———(1)

We know that, $x^2 + y^2 + \mathbf{2g}x + \mathbf{2f}y + \mathbf{c} = 0$ ———(2)

On comparing (1) & (2), we get

$2g = 4$	$2f = 4$	
$g = \frac{4}{2} = 2$	$f = \frac{4}{2} = 2$	$c = -1$

$\therefore$ Centre C = $(-g, -f)$ = **(–2, – 2)**

Radius r = $\sqrt{g^2 + f^2 - c} = \sqrt{(2)^2 + (2)^2 - (-1)}$ = 3 units

2. Show that the circles $x^2 + y^2 - 2x + 4y - 3 = 0$ and $x^2 + y^2 - 2x + 4y + 5 = 0$ are concentric circles

Answer

Given $x^2 + y^2 - 2x + 4y - 3 = 0$

$x^2 + y^2 - 2x + 4y + 5 = 0$

Here $c_1 = -3$, $c_2 = 5$ $\because c_1 \neq c_2$ $\therefore$ Given circles are concentric

3. If $\vec{a} = 3\vec{i} + 2\vec{j} + \vec{k}$ and $\vec{b} = \vec{i} + 3\vec{j} + \vec{k}$, find $3\vec{a} + \vec{b}$

Answer

$3\vec{a} + \vec{b} = 3(3\vec{i} + 2\vec{j} + \vec{k}) + (\vec{i} + 3\vec{j} + \vec{k})$

$= 9\vec{i} + 6\vec{j} + 3\vec{k} + \vec{i} + 3\vec{j} + \vec{k}$

$= 10\vec{i} + 9\vec{j} + 4\vec{k}$

4. What are the values of $\vec{i} \bullet \vec{j}$ and $\vec{k} \bullet \vec{k}$?

Answer

$\vec{i} \bullet \vec{j} = 0$, $\vec{k} \bullet \vec{k} = 1$

5. If $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 10$, find the angle between $\vec{a}$ and $\vec{b}$

Answer

Given $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{a} \times \vec{b}| = 10$

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$$

$$\sin\theta = \frac{10}{(3)(5)}$$

$$\sin\theta = \frac{2}{3}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{2}{3}\right)$$

6. Evaluate: $\int (x^2 + \cos x)\, dx$

Answer

$$\text{Given} \int (x^2 + \cos x) dx = \frac{x^3}{3} + \sin x + c$$

7. Evaluate: $\int \frac{1}{\sqrt{x^2+9}} dx$ {*Out of Syllabus*}

Answer

Formula: $\int \frac{1}{\sqrt{x^2+a^2}} dx = \log[x + \sqrt{x^2 + a^2}]$

$$\int \frac{1}{\sqrt{x^2+9}} dx = \int \frac{1}{\sqrt{x^2+3^2}} dx = \log[x + \sqrt{x^2 + 9}] + c$$

8. Evaluate: $\int_1^2 \left(x + x^2\right)dx$

Answer Refer *October 2016 ,Question no: 8 ,Page no:29*

PART - B

9. Prove that the equation $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ is a parabola

Answer Refer *October 2017 ,Question no: 9 ,Page no:79*

10. Show that the two vectors $\vec{i} - 3\vec{j} + 5\vec{k}$ and $-2\vec{i} + 6\vec{j} + 4\vec{k}$ are mutually perpendicular.

Answer Refer *October 2017 ,Question no: 10 ,Page no:80*

11. Find the area of the triangle whose adjacent sides are $2\vec{i} + 3\vec{j} - \vec{k}$ and $\vec{i} + 3\vec{j} + \vec{k}$

Answer

Let $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = \vec{i} + 3\vec{j} + \vec{k}$

Area of the triangle $= \frac{1}{2}\left|\vec{a} \times \vec{b}\right|$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & -1 \\ 1 & 3 & 1 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} 3 & -1 \\ 3 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix}$$

$$= \vec{i}(3 + 3) - \vec{j}(2 + 1) + \vec{k}(6 - 3)$$

$$\vec{a} \times \vec{b} = 6\vec{i} - 3\vec{j} + 3\vec{k}$$

$$\left|\vec{a} \times \vec{b}\right| = \sqrt{(6)^2 + (-3)^2 + (3)^2}$$

$$= \sqrt{54}$$

Area of the triangle $= \frac{1}{2}\sqrt{54}$ square units

12. Find the value of 'm' if the vectors $\vec{i} + 2\vec{j} - \vec{k}$, $2\vec{i} + m\vec{j} - 3\vec{k}$ and $3\vec{i} + \vec{j} + 4\vec{k}$ are coplanar

Answer Let $\vec{a} = \vec{i} + 2\vec{j} - \vec{k}$, $\vec{b} = 2\vec{i} + m\vec{j} - 3\vec{k}$

$\vec{c} = 3\vec{i} + \vec{j} + 4\vec{k}$

Condition for coplanar $[\vec{a}, \vec{b}, \vec{c}] = 0$

$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & m & -3 \\ 3 & 1 & 4 \end{vmatrix} = 0$$

$$1\begin{vmatrix} m & -3 \\ 1 & 4 \end{vmatrix} - 2\begin{vmatrix} 2 & -3 \\ 3 & 4 \end{vmatrix} - 1\begin{vmatrix} 2 & m \\ 3 & 1 \end{vmatrix} = 0$$

$$1(4m + 3) - 2(8 + 9) - 1(2 - 3m) = 0$$

$$4m + 3 - 16 - 18 - 2 + 3m = 0$$

$$7m - 33 = 0$$

$$7m = 33$$

$$m = \frac{33}{7}$$

13. Evaluate $\int_0^{\frac{\pi}{2}} \frac{cos^2x}{1+sinx}\, dx$

Answer

$$\int_0^{\frac{\pi}{2}} \frac{cos^2x}{1 + sinx} dx = \int_0^{\frac{\pi}{2}} \frac{1 - sin^2x}{1 + sinx} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{(1 + sinx)(1 - sinx)}{(1 + sinx)} dx$$

$$= \int_0^{\frac{\pi}{2}} (1 - sinx)dx$$

$$= \lfloor x + cosx \rfloor_0^{\frac{\pi}{2}}$$

$$= (\frac{\pi}{2} + cos\frac{\pi}{2}) - (0 + cos\,0)$$

$$= \frac{\pi}{2} - 1$$

14. Evaluate $\int \frac{e^x}{1+e^x} dx$

Answer Refer *April 2018 ,Question no:* 20 b(i) ,*Page no:*113

15. Evaluate: $\int x \log x \, dx$

Answer Refer *October 2017 ,Question no: 15 ,Page no:81*

16. Evaluate: $\int (x^2 + x + 1)(x + 5)\, dx$

Answer

$$\int (x^2 + x + 1)(x + 5)\, dx$$

$$= \int (x^3 + 5x^2 + x^2 + 5x + x + 5)\, dx$$

$$= \int (x^3 + 6x^2 + 6x + 5)\, dx$$

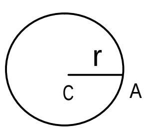
$$= \frac{x^4}{4} + \frac{6x^3}{3} + \frac{6x^2}{2} + 5x + c$$

$$= \frac{x^4}{4} + 2x^3 + 3x^2 + 5x + c$$

PART - C

17.(a). Find the equation of the circle passing through the point A (2, 3) and having its centre at C (4, 1)

Answer Given A = (2, 3),C = (4, 1)



Radius $AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here (x_1, y_1) = (2, 3) and (x_2, y_2) = (4, 1)

$$r = AC = \sqrt{(4 - 2)^2 + (1 - 3)^2}$$

$$= \sqrt{(2)^2 + (-2)^2}$$

$$= \sqrt{8} \text{ units}$$

Equation of circle is $(x - h)^2 + (y - k)^2 = r^2$

Here r = $\sqrt{8}$ and (h , k) = (4 ,1)

$$(x-\mathbf{4})^2 + (y-\mathbf{1})^2 = (\sqrt{8})^2$$

$$(x-\mathbf{4})(x-\mathbf{4}) + (y-\mathbf{1})(y-\mathbf{1}) = 8$$

$$x^2 - 4x - 4x + 16 + y^2 - y - y + 1 = 8$$

$$x^2 + y^2 - 8x - 2y + 17 - 8 = 0$$

$$x^2 + y^2 - 8x - 2y + \mathbf{9} = 0$$

which is the required equation of the circle

17.(b). Prove that the circles $x^2 + y^2 - 8x + 6y - 23 = 0$ and $x^2 + y^2 - 2x - 5y + 16 = 0$ cut orthogonally

Answer Refer *April 2016 ,Question no:* 17 b *,Page no:*10

17.(c). Show that the equation $3x^2 + 7xy + 2y^2 + 5x + 5y + 2 = 0$ represents a pair of straight lines

Answer

Given $\mathbf{3x^2 + 7xy + 2y^2 + 5x + 5y + 2 = 0}$ —— (1)

We know that, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ——(2)

On comparing(1) & (2) ,we get

a = 3	2h = 7	b = 2	2g = 5	2f = 5	c = 2
2a = 6		2b = 4			2c = 4

The condition for pair of straight line is $\begin{vmatrix} \mathbf{2a} & \mathbf{2h} & \mathbf{2g} \\ \mathbf{2h} & \mathbf{2b} & \mathbf{2f} \\ \mathbf{2g} & \mathbf{2f} & \mathbf{2c} \end{vmatrix} = \mathbf{0}$

$$\begin{vmatrix} 6 & 7 & 5 \\ 7 & 4 & 5 \\ 5 & 5 & 4 \end{vmatrix} = 0$$

$$6\begin{vmatrix} 4 & 5 \\ 5 & 4 \end{vmatrix} - 7\begin{vmatrix} 7 & 5 \\ 5 & 4 \end{vmatrix} + 5\begin{vmatrix} 7 & 4 \\ 5 & 5 \end{vmatrix} = 0$$

$$6[-9] - 7[3] + 5[15] = 0$$

$$-54 - 21 + 75 = 0$$

$$0 = 0$$

Represents a pair of straight lines

18.(a). The position vector of the points A , B and C are $4\vec{i} + 2\vec{j} + 3\vec{k}$ **,** $2\vec{i} + 3\vec{j} + 4\vec{k}$ **and** $3\vec{i} + 4\vec{j} + 2\vec{k}$ **. Prove that these points form an equilateral triangle**

Answer Refer *October 2017 ,Question no:* 18(a) *,Page no:*86

18.(b). If $\vec{a} = 2\vec{i} + \vec{j} + 3\vec{k}$, ***and*** $\vec{b} = \vec{i} - 4\vec{j} - 6\vec{k}$ **, find the projection of** $\vec{a}$ **on** $\vec{b}$ **. Also find the angle between the two vectors** $\vec{a}$ **and** $\vec{b}$**.**

Answer Refer *April 2018 ,Question no:* 18(b) *,Page no:*109

18.(c) Find the work done by the force $\vec{i} + 3\vec{j} - \vec{k}$ **when it displaces a particle from the point** $2\vec{i} - 6\vec{j} + 7\vec{k}$ **to the point** $3\vec{i} - \vec{j} - 5\vec{k}$

Answer

Formula: Work done W = $\vec{F} \bullet \vec{d}$

$\vec{F} = \vec{i} + 3\vec{j} - \vec{k}$

$\vec{d}$ = ***To the point – From the point***

$= (3\vec{i} - \vec{j} - 5\vec{k}) - (2\vec{i} - 6\vec{j} + 7\vec{k})$

$= 3\vec{i} - \vec{j} - 5\vec{k} - 2\vec{i} + 6\vec{j} - 7\vec{k}$

$= \vec{i} + 5\vec{j} - 12\vec{k}$

Work done W = $\vec{F} \bullet \vec{d}$

$$= (\vec{i} + 3\vec{j} - \vec{k}) \bullet (\vec{i} + 5\vec{j} - 12\vec{k})$$

$$= (1)(1) + (3)(5) + (-1)(-12)$$

$$= 1 + 15 + 12$$

Work done W = 28 units

19.(a). Find the unit vector perpendicular to each of the vectors $3\vec{i} + 3\vec{j} + \vec{k}$ and $2\vec{i} - 5\vec{j} + 3\vec{k}$.

Answer

Let $\vec{a} = 3\vec{i} + 3\vec{j} + \vec{k}$, $\vec{b} = 2\vec{i} - 5\vec{j} + 3\vec{k}$,

Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ is $\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 3 & 1 \\ 2 & -5 & 3 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} 3 & 1 \\ -5 & 3 \end{vmatrix} - \vec{j}\begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} + \vec{k}\begin{vmatrix} 3 & 3 \\ 2 & -5 \end{vmatrix}$$

$$= \vec{i}(9 + 5) - \vec{j}(9 - 2) + \vec{k}(-15 - 6)$$

$$\vec{a} \times \vec{b} = 14\vec{i} - 7\vec{j} - 21\vec{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(14)^2 + (-7)^2 + (-21)^2}$$

$$= \sqrt{196 + 49 + 441}$$

$$= \sqrt{686}$$

$$\therefore \hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

$$= \frac{14\vec{i} - 7\vec{j} - 21\vec{k}}{\sqrt{686}}$$

19.(b). Find the moment of the force $6\vec{i} + \vec{j} + \vec{k}$ acting through a point $\vec{i} + 2\vec{j} + 3\vec{k}$ about the point $-\vec{i} - \vec{j} + \vec{k}$

Answer Formula: Moment of the force $= \vec{r} \times \vec{F}$

$\vec{F} = 6\vec{i} + \vec{j} + \vec{k}$

$\vec{r}$ = *Acting point – About point*

$= (\vec{i} + 2\vec{j} + 3\vec{k}) - (-\vec{i} - \vec{j} + \vec{k})$

$= \vec{i} + 2\vec{j} + 3\vec{k} + \vec{i} + \vec{j} - \vec{k}$

$= 2\vec{i} + 3\vec{j} + 2\vec{k}$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 2 \\ 6 & 1 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 2 \\ 6 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 3 \\ 6 & 1 \end{vmatrix}$$

$$= \vec{i}(3 - 2) - \vec{j}(2 - 12) + \vec{k}(2 - 18)$$

$$= \vec{i}(1) - \vec{j}(-10) + \vec{k}(-16)$$

$$\vec{r} \times \vec{F} = \vec{i} + 10\vec{j} - 16\vec{k}$$

19.(c). If $\vec{a} = 2\vec{i} - \vec{j} + 3\vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} - \vec{k}$, $\vec{c} = 3\vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{d} = \vec{i} + 3\vec{j} + 4\vec{k}$ find $(\vec{a} \times \vec{b}) \bullet (\vec{c} \times \vec{d})$

Answer Given $\vec{a} = 2\vec{i} - \vec{j} + 3\vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} - \vec{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 3 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} -1 & 3 \\ 2 & -1 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}$$

$$= \vec{i}(1 - 6) - \vec{j}(-2 - 3) + \vec{k}(4 + 1)$$

$$= \vec{i}(-5) - \vec{j}(-5) + \vec{k}(5)$$

$$\vec{a} \times \vec{b} = -5\vec{i} + 5\vec{j} + 5\vec{k}$$

Given $\vec{c} = 3\vec{i} + 2\vec{j} + 3\vec{k}$, $\vec{d} = \vec{i} + 3\vec{j} + 4\vec{k}$

$$\vec{\mathbf{c}} \times \vec{\mathbf{d}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & 3 \\ 1 & 3 & 4 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} - \vec{j}\begin{vmatrix} 3 & 3 \\ 1 & 4 \end{vmatrix} + \vec{k}\begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix}$$

$$= \vec{i}(8 - 9) - \vec{j}(12 - 3) + \vec{k}(9 - 2)$$

$$= \vec{i}(-1) - \vec{j}(9) + \vec{k}(7)$$

$$\vec{c} \times \vec{d} = -\vec{i} - 9\vec{j} + 7\vec{k}$$

$$(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \bullet (\vec{\mathbf{c}} \times \vec{\mathbf{d}})$$

$$= (-5\vec{i} + 5\vec{j} + 5\vec{k}) \bullet (-\vec{i} - 9\vec{j} + 7\vec{k})$$

$$= (-5)(-1) + (5)(-9) + (5)(7)$$

$$= 5 - 45 + 35 = -5$$

20.(a). Evaluate

(i) $\int (x - 1)(2x + 3)\, dx$ **(ii)** $\int 2 \sin 3x \cos x\, dx$

Answer

(i) $\int (x - 1)(2x + 3)\, dx = \int (2x^2 + 3x - 2x - 3)\, dx$

$$= \int (2x^2 + x - 3)\, dx$$

$$= \frac{2x^3}{3} + \frac{x^2}{2} - 3x + c$$

$$(ii) \int 2 \sin 3x \cos x\, dx = \int \sin(3x + x) + \sin(3x - x)\, dx$$

$$= \int (\sin 4x + \sin 2x)\, dx$$

2 sinA cos B = sin (A + B) + sin(A –B)

$$= \left[\frac{-\cos 4x}{4} - \frac{\cos 2x}{2}\right] + c$$

20.(b). Evaluate (i) $\int \frac{6x^2-1}{2x^3-x+5}\,dx$ (ii) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Answer Refer *October 2016 ,Question no:* 20(b) *,Page no:*44

20.(c). Evaluate (i) $\int \frac{dx}{16+x^2}$ (ii) $\int \frac{dx}{\sqrt{36-x^2}}$

(i)Answer Refer *October 2017 ,Question no:* 20(c) *,Page no:*93

(ii):

$$\int \frac{dx}{\sqrt{a^2-x^2}} = sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{dx}{\sqrt{36-x^2}} = \int \frac{dx}{\sqrt{(6)^2-(x)^2}}$$

$$= sin^{-1}\left(\frac{x}{6}\right) + c$$

21.(a). Evaluate (i) $\int x\, e^{4x}\,dx$ (ii) $\int x \sin 2x\,dx$

Answer

(i) $\int x\, e^{4x}\,dx$

Let $u = x$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$\int dv = \int e^{4x}dx$$

$$v = \frac{e^{4x}}{4}$$

Formula: $\int udv = uv - \int v\,du$

$$\int x\, e^{4x}\,dx = x\left(\frac{e^{4x}}{4}\right) - \frac{1}{4}\int e^{4x}dx = \frac{x\,e^{4x}}{4} - \frac{1}{4}\left(\frac{e^{4x}}{4}\right) + c$$

$$= \frac{x\,e^{4x}}{4} - \frac{e^{4x}}{16} + c$$

(ii)Answer Refer *October 2017 ,Question no:* 21(a) *,Page no:*94

21.(b). Evaluate: (i) $\int x^2 e^x dx$ (ii) $\int x^2 \cos 3x \, dx$

Answer

(i) $\int x^2 e^x dx$

Let $u = x^2$	$\int dv = \int e^x dx$
$u' = 2x$	$v = e^x$
$u'' = 2$	$v_1 = e^x$
	$v_2 = e^x$

Formula: $\int u\, dv = uv - u'v_1 + u''v_2 - \cdots$

$\int x^2 e^x dx = x^2 e^x - 2xe^x + 2e^x + c$

(ii)Answer Refer *October 2016 ,Question n<u>o</u>:* 21(b) *,Page n<u>o</u>:*46

21.(c). Evaluate: $\int_0^{\frac{\pi}{4}} \tan x \sec^2 x \, dx$

Answer

$\int_0^{\frac{\pi}{4}} \tan x \sec^2 x \, dx$ ---- (1)

Put $t = \tan x$	$(1) \Rightarrow \int_0^1 t\, dt$
$\frac{dt}{dx} = \sec^2 x$	$= \left[\frac{t^2}{2}\right]_0^1$
$dt = \sec^2 x dx$	$= \frac{1^2 - 0^2}{2}$
Limits:	$= \frac{1}{2}$
When $x = 0$,	
$t = \tan 0 = 0$	
When $x = \frac{\pi}{4}$,	
$t = \tan\frac{\pi}{4} = 1$	