# APRIL 2017

$Time - Three\ hours$

$(Maximum\ Marks: 75)$

*[ N.B:- (1) Answer any FIVE questions in each of PART-A& PART-B and any two divisions of each questions in PART-C*

*(2) Each questions carries 2(two)marks in PART-A ,3(three)marks in PART-B and 5(five)marks for each division in PART-C.]*

## PART – A

1. Find the centre and radius of the circle $x^2 + y^2 + 10x + 8y + 5 = 0$.
2. Show that the equation $7x^2 + 3xy + 2y^2 - x + 2y - 1 = 0$ represents an ellipse
3. Find the projection of the vector $2\vec{i} + 3\,\vec{j} + \vec{k}$ on the vector $3\vec{i} - \vec{j} + \vec{k}$.
4. Evaluate: $[\,\vec{i} + \vec{j}, \vec{j} + \vec{k}, \vec{k} + \vec{i}\,]$
5. If $|\vec{a}| = 6$, $|\vec{b}| = 4$, $\vec{a} \bullet \vec{b} = 12$, find the angle between them.
6. Evaluate $\int \frac{dx}{x\, \log x}$
7. Evaluate: $\int \frac{dx}{\sqrt{9-x^2}}$

8 Evaluate : $\int_0^1 (3x^2 - 2x + 7)\,dx$

## PART – B

9. Show that $x^2 + y^2 - 2x + 6y + 6 = 0$ and $x^2 + y^2 - 5x + 6y + 15 = 0$ circles touch each other.

10. Show that the points with position vectors $2\vec{i} - \vec{j} + 3\vec{k}$, $3\vec{i} - 5\vec{j} + \vec{k}$ and $-\vec{i} + 11\vec{j} + 9\vec{k}$ are collinear.

11. Find the area of the parallelogram whose adjacent sides are $3\vec{i} - \vec{k}$ and $\vec{i} + \vec{j} + \vec{k}$.

12. Prove that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$

13. Evaluate: $\int \cos^3 7x \, dx$

14. Evaluate: $\int \sqrt{1 + \sin 2x} \, dx$

15. Evaluate: $\int x^2 \sin 3x \, dx$

16. Evaluate: $\int_0^{\frac{\pi}{2}} (2 + \sin x)^2 \cos x \, dx$

## PART- C

17.(a) Find the equation of the circle, two of whose diameters are $2x - 3y + 1 = 0$ and $x + 2y - 17 = 0$ and radius is 8 units

(b) Find the equation of the circle concentric with the circle $x^2 + y^2 - 6x + 10y - 1 = 0$ and passing through the point (1, 1)

(c) For the quadratic equation $2x^2 + 7xy + 3y^2 + 13x - y - 24 = 0$, Identify the conic

18.(a) Show that the points with position vectors $3\vec{i} - \vec{j} + 6\vec{k}$

$5\vec{i} - 2\vec{j} + 7\vec{k}$ and $6\vec{i} - 5\vec{j} + 2\vec{k}$ form a right angled triangle

(b) Show that $(\vec{a} \bullet \vec{i})\vec{i} + (\vec{a} \bullet \vec{j})\vec{j} + (\vec{a} \bullet \vec{k})\vec{k} = \vec{a}$,

*if* $\vec{a}$ is any vector

(c) The forces $2\vec{i} - 5\vec{j} + 6\vec{k}$, $-\vec{i} + 2\vec{j} - \vec{k}$ and $2\vec{i} + 7\vec{j}$ act on a particle and displace it from the point $4\vec{i} - 3\vec{j} - 2\vec{k}$ to the point $6\vec{i} + \vec{j} - 3\vec{k}$. Find the total work done by the forces.

19.(a) Find the unit vector perpendicular to two vectors $\vec{i} - \vec{j} + 3\vec{k}$ *and* $2\vec{i} + 3\vec{j} - \vec{k}$. Also find the angle between them.

(b) Find the magnitude of the torque about the point (4, 3, − 1) of the force represented by $6\vec{i} + \vec{j} - \vec{k}$ acting through the point (0 , 1 , − 1)

(c) If $\vec{a} = 2\vec{i} + 3\vec{j} + \vec{k}$, $\vec{b} = \vec{i} - 2\vec{j} + 3\vec{k}$ *and* $\vec{c} = 3\vec{i} + 2\vec{j} - 5\vec{k}$ verify that

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \bullet \vec{c})\vec{b} - (\vec{a} \bullet \vec{b})\vec{c}$$

20.(a) Evaluate: (i) $\int \frac{\sin^2 x}{1+\cos x} dx$ (ii) $\int (\tan x + \cot x)^2 dx$

(b) Evaluate: (i) $\int \frac{(x+1)dx}{x^2+2x-1}$ (ii) $\int \frac{\sec^2 x}{5+\tan x} dx$

(c) Evaluate: (i) $\int \frac{dx}{25-9x^2}$ (ii) $\int \frac{dx}{(2x+3)^2+9}$

21.(a) Evaluate: $\int x^2 \log x \, dx$ (ii) $\int x \cos 2x \, dx$

(b) Evaluate: $\int x^2 e^{-3x} dx$ (ii) $\int x^2 \sin 6x \, dx$

(c) Evaluate: $\int_0^{\frac{\pi}{2}} \log(\tan x)dx$

## ANSWERS

## PART - A

**1. Find the centre and radius of the circle**

**$x^2 + y^2 + 10x + 8y + 5 = 0$**

***Answer***

Given $x^2 + y^2 + 10x + 8y + 5 = 0$ ———(1)

We know that, $x^2 + y^2 + \mathbf{2gx} + \mathbf{2fy} + \mathbf{c} = 0$ ———(2)

On comparing (1) & (2),we get

| | | |
|---|---|---|
| $2g = 10$ | $2f = 8$ | |
| $g = \frac{10}{2} = 5$ | $f = \frac{8}{2} = 4$ | c = 5 |

$\therefore$ Centre C = $(-g, -f) = \mathbf{(-5, -4)}$

Radius r = $\sqrt{g^2 + f^2 - c}$

$r = \sqrt{(5)^2 + (4)^2 - (5)}$

$= \sqrt{25 + 16 - 5} = \sqrt{36}$ = **6 units**

**2. Show that the equation $7x^2 + 3xy + 2y^2 - x + 2y - 1 = 0$ represents an ellipse**

***Answer***

Given $7x^2 + 3xy + 2y^2 - x + 2y - 1 = 0$ —— (1)

We know that, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ —— (2)

On comparing (1) &(2),we get a = 7, b = 2 , 2h = 3 ⇒ $h = \frac{3}{2}$

Condition for an ellipse $\mathbf{h^2 - ab < 0}$

$$h^2 - ab = \left(\frac{3}{2}\right)^2 - (7)(2)$$

$$= \left(\frac{9}{4}\right) - 14$$

$$= \frac{9-56}{4} = \frac{-47}{4} < \mathbf{0}$$

∴ The given conic is **an ellipse**



**3. Find the projection of the vector $2\vec{i} + 3\vec{j} + \vec{k}$ on the vector $3\vec{i} - \vec{j} + \vec{k}$**

***Answer*** Let $\vec{a} = 2\vec{i} + 3\vec{j} + \vec{k}$ , $\vec{b} = 3\vec{i} - \vec{j} + \vec{k}$

$$\vec{a} \bullet \vec{b} = (2\vec{i} + 3\vec{j} + \vec{k}) \bullet (3\vec{i} - \vec{j} + \vec{k})$$

$$= (2)(3) + (3)(-1) + (1)(1)$$

$$= 6 - 3 + 1 = 4$$

$$|\vec{b}| = \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{(3)^2 + (-1)^2 + (1)^2} = \sqrt{9 + 1 + 1} = \sqrt{\mathbf{11}}$$

Formula: Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \bullet \vec{b}}{|\vec{b}|}$

$$= \frac{4}{\sqrt{11}} \text{ units}$$

**4. Evaluate:** $[\vec{\imath} + \vec{\jmath}, \vec{\jmath} + \vec{k}, \vec{k} + \vec{\imath}]$

***Answer Refer*** *October 2016 ,Question no: 5 ,Page no:28*

**5. If** $|\vec{a}| = 6, |\vec{b}| = 4, \vec{a} \bullet \vec{b} = 12$**, find the angle between them**

***Answer***

Given $|\vec{a}| = 6, |\vec{b}| = 4, \vec{a} \bullet \vec{b} = 12$

$$\cos\theta = \frac{\vec{a} \bullet \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\Rightarrow \cos\theta = \frac{12}{(6)(4)}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \theta = 60^0$$

**6. Evaluate:** $\int \frac{dx}{x\,logx}$

***Answer***

$\int \frac{dx}{x\,logx}$ ----- (1)

Put $t = logx$

$\frac{dt}{dx} = \frac{1}{x}$

$dt = \frac{1}{x}\,dx$

$$\int \frac{dx}{x\,logx} = \int \frac{1}{t}\,dt$$

$$= \log(t) + c$$

$$= \log(logx) + c$$

**7. Evaluate:** $\int \frac{dx}{\sqrt{9-x^2}}$

***Answer***

$$\int \frac{dx}{\sqrt{9-x^2}} = \int \frac{dx}{\sqrt{3^2-x^2}} = sin^{-1}\left(\frac{x}{3}\right) + c$$

**8. Evaluate : $\int_0^1 (3x^2 - 2x + 7)dx$**

***Answer***

$$\int_0^1 (3x^2 - 2x + 7)dx = \left[\frac{3x^3}{3} - \frac{2x^2}{2} + 7x\right]_0^1$$

$$= [x^3 - x^2 + 7x]_0^1$$

$$= \left(1^3 - 1^2 + 7(1)\right) - 0$$

$$= (1 - 1 + 7) = 7$$

## PART - B

**9. Show that $x^2 + y^2 - 2x + 6y + 6 = 0$ and**

**$x^2 + y^2 - 5x + 6y + 15 = 0$ circles touch each other**

***Answer***

**Step 1:** Given: $x^2 + y^2 - 2x + 6y + 6 = 0$ ——(1)

$x^2 + y^2 + 2gx + 2fx + c = 0$ —— (2)

On comparing (1) & (2), we get

$$2g = -2 \qquad g = -\frac{2}{2} = -1$$

$$2f = 6 \qquad f = \frac{6}{2} = 3$$

$$c = 6$$

$\therefore$ Centre C = (–g, –f ) ⇒ $C_1 = (1, -3)$

Radius r $= \sqrt{g^2 + f^2 - c}$

$$r_1 = \sqrt{(-1)^2 + (3)^2 - 6} = \sqrt{1 + 9 - 6} = \sqrt{4} = 2\,units$$

**Step 2:** Given: $x^2 + y^2 - 5x + 6y + 15 = 0$ ——— (3)

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \text{———(2)}$$

On comparing (3) & (2), we get

| | | |
|---|---|---|
| $2g = -5$ <br> $g = \frac{-5}{2} = -2.5$ | $2f = 6$ <br> $f = \frac{6}{2} = 3$ | c = 15 |

$\therefore$ Centre C = (–g, –f ) ⇒ **$C_2$** = ( 2.5 , –3)

Radius r $= \sqrt{g^2 + f^2 - c}$

$$r_2 = \sqrt{(-2.5)^2 + (3)^2 - 15}$$

$$= \sqrt{6.25 + 9 - 15} = \sqrt{0.25} = 0.5\,units$$

**Step 3:** **$d = C_1C_2 =$** $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here $C_1 = (1, -3) = (x_1, y_1)$, $C_2 = (2.5, -3) = (x_2, y_2)$

$$d = \sqrt{(2.5 - 1)^2 + (-3 + 3)^2}$$

$$= \sqrt{(1.5)^2 + (0)^2} = \sqrt{2.25} = 1.5\ units$$

$$\therefore r_1 - r_2 = 2 - 0.5 = 1.5 = d$$

$$\therefore d = r_1 - r_2$$

$\therefore$ The circles touch each other internally

**10. Show that the points with position vectors $2\vec{\imath} - \vec{\jmath} + 3\vec{k}$, $3\vec{\imath} - 5\vec{\jmath} + \vec{k}$ and $-\vec{\imath} + 11\vec{\jmath} + 9\vec{k}$ are collinear**

***Answer***

Let $\overrightarrow{OA} = 2\vec{i} - \vec{j} + 3\vec{k}$

$\overrightarrow{OB} = 3\vec{i} - 5\vec{j} + \vec{k}$

$\overrightarrow{OC} = -\vec{i} + 11\vec{j} + 9\vec{k}$

$\boldsymbol{\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}}$

$\overrightarrow{AB} = (3\vec{i} - 5\vec{j} + \vec{k}) - (2\vec{i} - \vec{j} + 3\vec{k})$

$\overrightarrow{AB} = 3\vec{i} - 5\vec{j} + \vec{k} - 2\vec{i} + \vec{j} - 3\vec{k}$

$\boldsymbol{\overrightarrow{AB} = \vec{i} - 4\vec{j} - 2\vec{k}}$ **------ ( 1)**

$\boldsymbol{\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}}$

$\overrightarrow{BC} = (-\vec{i} + 11\vec{j} + 9\vec{k}) - (3\vec{i} - 5\vec{j} + \vec{k})$

$\overrightarrow{BC} = -\vec{i} + 11\vec{j} + 9\vec{k} - 3\vec{i} + 5\vec{j} - \vec{k}$

$\boldsymbol{\overrightarrow{BC} = -4\vec{i} + 16\vec{j} + 8\vec{k}}$ **-------(2)**

$\boldsymbol{\overrightarrow{BC}} = -4(\vec{i} - 4\vec{j} - 2\vec{k})$

$\boldsymbol{\overrightarrow{BC}} = -4(\boldsymbol{\overrightarrow{AB}})$ **{From(1) }**

$\Rightarrow \overrightarrow{AB} \parallel \overrightarrow{BC}$, Hence A,B, C are collinear points

**11. Find the area of the parallelogram whose adjacent sides are $\boldsymbol{3\vec{i} - \vec{k}}$ *and* $\boldsymbol{\vec{i} + \vec{j} + \vec{k}}$**

***Answer*** Let $\vec{a} = 3\vec{i} - \vec{k}$, $\vec{b} = \vec{i} + \vec{j} + \vec{k}$

Area of the parallelogram $= |\vec{a} \times \vec{b}|$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 0 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 3 & 0 \\ 1 & 1 \end{vmatrix}$$

$$= \vec{i}(0 + 1) - \vec{j}(3 + 1) + \vec{k}(3 - 0)$$

$$= \vec{i}(1) - \vec{j}(4) + \vec{k}(3)$$

$$\vec{a} \times \vec{b} = \vec{i} - 4\,\vec{j} + 3\,\vec{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{(1)^2 + (-4)^2 + (3)^2}$$

$$= \sqrt{1 + 16 + 9} = \sqrt{26} \text{ square units}$$

**12. Prove that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$**

***Answer***

**LHS** $= \vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b})$

$= (\vec{a} \times \vec{a}) + (\vec{b} \times \vec{b}) + (\vec{c} \times \vec{c})$

$= (\vec{0}) + (\vec{0}) + (\vec{0}) = \vec{0}$ = ***RHS***

**13. Evaluate: $\int \cos^3 7x\, dx$**

***Answer***

Formula: $\cos^3 A = \frac{\cos 3A + 3\cos A}{4}$

$Here\ A = 7x \quad \cos^3 7x = \frac{\cos 3(7x) + 3\cos 7x}{4}$

$$\cos^3 7x = \frac{\cos 21x + 3\cos 7x}{4}$$

$$\int \cos^3 7x\, dx = \int \frac{\cos 21x + 3\cos 7x}{4}\, dx$$

$$= \frac{1}{4}\int (cos21x + 3cos7x)\ dx$$

$$= \frac{1}{4}\left[\frac{sin21x}{21} + \frac{3sin7x}{7}\right] + c$$

**14. Evaluate: $\int \sqrt{1 + sin2x}\ dx$**

***Answer***

$$\int \sqrt{1 + sin2x}\ dx$$

$$= \int \sqrt{(sin^2x + cos^2x + 2sinx\ cosx)}dx$$

$$= \int \sqrt{(sinx + cosx)^2}dx$$

$$= \int (sinx + cosx\ )dx \quad = -cosx + sinx + c$$

**15. Evaluate: $\int x^2 \sin 3x\ dx$**

***Answer Refer*** *April 2016 ,Question no:* 21(b)(i)*,Page no:*21

**16. Evaluate: $\int_0^{\frac{\pi}{2}} (2 + \text{sinx})^3 \text{cosx}\ \ \text{dx}$**

***Answer Refer*** *April 2016 ,Question no:* 21(c)*,Page no:*22

## PART - C

**17.(a). Find the equation of the circle, two of whose diameters are $2x - 3y + 1 = 0$ and $x + 2y - 17 = 0$ and radius is 8 units**

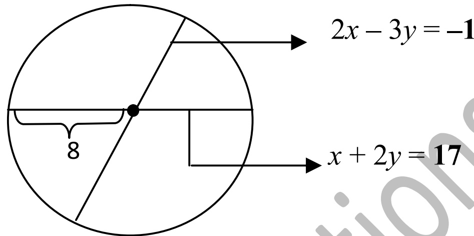
***Answer***

Given $2x - 3y + 1 = 0$

$$\Rightarrow \mathbf{2x - 3y = -1}$$

and $x + 2y - 17 = 0$

$$\Rightarrow \mathbf{x + 2y = 17}$$




By cramer's Rule

$$\Delta = \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} = (2)(2) - (1)(-3) = 4 + 3 = 7$$

$$\Delta_x = \begin{vmatrix} -1 & -3 \\ 17 & 2 \end{vmatrix} = (-1)(2) - (17)(-3) = -2 + 51 = 49$$

$$\Delta_y = \begin{vmatrix} 2 & -1 \\ 1 & 17 \end{vmatrix} = (2)(17) - (1)(-1) = 34 + 1 = 35$$

$$x = \frac{\Delta_x}{\Delta} = \frac{49}{7} = 7$$

$$y = \frac{\Delta_y}{\Delta} = \frac{35}{7} = 5$$

$\therefore$ Centre C = $(h, k) = (7, 5)$ , Radius r = 8 units [given]

**Equation of circle is $(x - h)^2 + (y - k)^2 = r^2$**

$$(x - 7)^2 + (y - 5)^2 = 8^2$$

$$[x^2 + 7^2 - 2(x)(7)] + [y^2 + 5^2 - 2(y)(5)] = 64$$

$$x^2 + 49 - 14x + y^2 + 25 - 10y - 64 = 0$$

$$x^2 + y^2 - 14x - 10y + 74 - 64 = 0$$

$$x^2 + y^2 - 14x - 10y + 10 = 0$$

which is the required equation of the circle

**17.(b). Find the equation of the circle concentric with the circle $x^2 + y^2 - 6x + 10y - 1 = 0$ and passing through the point (1, 1)**

***Answer***

Given $x^2 + y^2 - 6x + 10y - 1 = 0$

Concentric circle is $x^2 + y^2 - 6x + 10y + k = 0$ —— (1)

(1) passes through (1, 1) ⇒ put $x = 1, y = 1$ in (1), we get

$$(1)^2 + (1)^2 - 6(1) + 10(1) + k = 0$$

$$1 + 1 - 6 + 10 + k = 0$$

$$6 + k = 0$$

$$\Rightarrow \mathbf{k = -6}$$

(1)⇒ Concentric circle is $x^2 + y^2 - 6x + 10y - \mathbf{6} = 0$

**17.(c). For the quadratic equation**

**$2x^2 + 7xy + 3y^2 + 13x - y - 24 = 0$, Identify the conic**

***Answer*** Given $2x^2 + 7xy + 3y^2 + 13x - y - 24 = 0$ —— (1)

We know that, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ —— (2)

On comparing (1) &(2), we get a =2, b = 3, $2h = 7 \Rightarrow$ ? $= \frac{7}{2}$

$$h^2 - ab = \left(\frac{7}{2}\right)^2 - (2)(3)$$

$$= \left(\frac{49}{4}\right) - 6$$

$$= \frac{49 - 24}{4}$$

$$= \frac{25}{4} > \mathbf{0}$$

$\therefore$ **The given conic is a hyperbola**

Given $2x^2 + 7xy + 3y^2 + 13x - y - 24 = 0$ —— (1)

We know that, $ax^2 + 2h\,xy + b\,y^2 + 2g\,x + 2f\,y + c = 0$ ——(2)

On comparing(1) & (2) ,we get

| | | | | | |
|---|---|---|---|---|---|
| a = 2 | 2h = 7 | b = 3 | 2g = 13 | 2f = –1 | c = – 24 |
| 2a = 4 | | 2b = 6 | | | 2c = – 48 |

The condition for pair of straight line is

$$\therefore \begin{vmatrix} \mathbf{2a} & \mathbf{2h} & \mathbf{2g} \\ \mathbf{2h} & \mathbf{2b} & \mathbf{2f} \\ \mathbf{2g} & \mathbf{2f} & \mathbf{2c} \end{vmatrix} = 0$$

$$\begin{vmatrix} 4 & 7 & 13 \\ \mathbf{7} & 6 & -1 \\ 13 & -1 & -48 \end{vmatrix} = 0$$

$$4\begin{vmatrix} 6 & -1 \\ -1 & -48 \end{vmatrix} - 7\begin{vmatrix} 7 & -1 \\ 13 & -48 \end{vmatrix} + 13\begin{vmatrix} 7 & 6 \\ 13 & -1 \end{vmatrix} = 0$$

$$4(-288-1) - 7(-336+13) + 13(-7-78) = 0$$

$$4(-289) - 7(-323) + 13(-85) = 0$$

$$-1156 + 2261 - 1105 = 0$$

$$-2261 + 2261 = 0$$

$$0 = 0$$

⇚ The given equation represents a **pair of straight line**



**18.(a). Show that the points with position vectors**

$$\mathbf{3\vec{i} - \vec{j} + 6\vec{k}\ ,\ 5\vec{i} - 2\vec{j} + 7\vec{k}\ \text{and}\ 6\vec{i} - 5\vec{j} + 2\vec{k}}$$

**form a right angled triangle**

***Answer Refer*** *October 2016 ,Question no:* 18(a),*Page no:*37

**18.(b). Show that** $(\vec{a}\bullet\vec{i})\vec{i} + (\vec{a}\bullet\vec{j})\vec{j} + (\vec{a}\bullet\vec{k})\vec{k} = \vec{a}$,

**If $\vec{a}$ is any vector**

***Answer*** Let $\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}$

$(\vec{a} \bullet \vec{i}) = (x\vec{i} + y\vec{j} + z\vec{k}) \bullet \vec{i}$

$(\vec{a} \bullet \vec{i}) = x\ (\vec{i} \bullet \vec{i}) + y\ (\vec{j} \bullet \vec{i}) + z\ (\vec{k} \bullet \vec{i})$

$(\vec{a} \bullet \vec{i}) = x\ (1) + y\ (0) + z\ (0)$

$\boldsymbol{(\vec{a} \bullet \vec{i}) = x}$

$(\vec{a} \bullet \vec{j}) = (x\vec{i} + y\vec{j} + z\vec{k}) \bullet \vec{j}$

$(\vec{a} \bullet \vec{j}) = x\ (\vec{i} \bullet \vec{j}) + y\ (\vec{j} \bullet \vec{j}) + z\ (\vec{k} \bullet \vec{j})$

$(\vec{a} \bullet \vec{j}) = x\ (0) + y\ (1) + z\ (0)$

$\boldsymbol{(\vec{a} \bullet \vec{j}) = y}$

$(\vec{a} \bullet \vec{k}) = (x\vec{i} + y\vec{j} + z\vec{k}) \bullet \vec{k}$

$(\vec{a} \bullet \vec{k}) = x\ (\vec{i} \bullet \vec{k}) + y\ (\vec{j} \bullet \vec{k}) + z\ (\vec{k} \bullet \vec{k})$

$(\vec{a} \bullet \vec{k}) = x\ (0) + y\ (0) + z\ (1)$

$\boldsymbol{(\vec{a} \bullet \vec{k}) = z}$

**L.H.S** $= (\vec{a} \bullet \vec{i})\ \vec{i} + (\vec{a} \bullet \vec{j})\ \vec{j} + (\vec{a} \bullet \vec{k})\ \vec{k}$

$= x\ \vec{i} + y\ \vec{j} + z\ \vec{k} = \vec{a}$ **= R.H.S**

**18.(c) The forces $2\,\vec{i} - 5\vec{j} + 6\vec{k}$, $-\vec{i} + 2\vec{j} - \vec{k}$ and $2\vec{i} + 7\vec{j}$ act on a particle and displace it from the point $4\vec{i} - 3\vec{j} - 2\vec{k}$ to the point $6\vec{i} + \vec{j} - 3\vec{k}$. Find the total work done by the forces.**

***Answer*** Formula: Work done W = $\vec{F} \bullet \vec{d}$

$\vec{F} = \vec{f}_1 + \vec{f}_2 + \vec{f}_3$

$= (2\vec{i} - 5\vec{j} + 6\vec{k}) + (-\vec{i} + 2\vec{j} - \vec{k}) + (2\vec{i} + 7\vec{j})$

$= 3\vec{i} + 4\vec{j} + 5\vec{k}$

$\vec{d}$ = *To the point – From the point*

$= (6\vec{i} + \vec{j} - 3\vec{k}) - (4\vec{i} - 3\vec{j} - 2\vec{k})$

$= 6\vec{i} + \vec{j} - 3\vec{k} - 4\vec{i} + 3\vec{j} + 2\vec{k}$

$= 2\vec{i} + 4\vec{j} - \vec{k}$

Work done W = $\vec{F} \bullet \vec{d}$

$= (3\vec{i} + 4\vec{j} + 5\vec{k}) \bullet (2\vec{i} + 4\vec{j} - \vec{k})$

$= (3)(2) + (4)(4) + (5)(-1)$

$= 6 + 16 - 5$

Work done **W = 17** units

**19.(a). Find the unit vector perpendicular to two vectors $\vec{i} - \vec{j} + 3\vec{k}$ *and* $2\vec{i} + 3\vec{j} - \vec{k}$ .Also find the angle between them**

***Answer*** Let $\vec{a} = \vec{i} - \vec{j} + 3\vec{k}$ , $\vec{b} = 2\vec{i} + 3\vec{j} - \vec{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 3 \\ 2 & 3 & -1 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} -1 & 3 \\ 3 & -1 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix}$$

$$= \vec{i}(1 - 9) - \vec{j}(-1 - 6) + \vec{k}(3 + 2)$$

$$= \vec{i}(-8) - \vec{j}(-7) + \vec{k}(5)$$

$$\vec{a} \times \vec{b} = -8\vec{i} + 7\vec{j} + 5\vec{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{x^2 + y^2 + z^2} \; \{x = -8, y = 7, z = 5\}$$

$$= \sqrt{(-8)^2 + (7)^2 + (5)^2}$$

$$= \sqrt{64 + 49 + 25}$$

$$|\vec{a} \times \vec{b}| = \sqrt{138}$$

$$|\vec{a}| = \sqrt{x^2 + y^2 + z^2} \; \{x = 1, y = -1, z = 3\}$$

$$= \sqrt{(1)^2 + (-1)^2 + (3)^2}$$

$$= \sqrt{1 + 1 + 9}$$

$$|\vec{a}| = \sqrt{11}$$

$$|\vec{b}| = \sqrt{x^2 + y^2 + z^2} \; \{x = 2, y = 3, z = -1\}$$

$$= \sqrt{(2)^2 + (3)^2 + (-1)^2}$$

$$= \sqrt{4 + 9 + 1}$$

$$|\vec{b}| = \sqrt{14}$$

∴The unit vector perpendicular to both $\vec{a}$ *and* $\vec{b}$ *is*

$$\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{-8\vec{i} + 7\vec{j} + 5\vec{k}}{\sqrt{138}}$$

∴Sine of the angle between $\vec{a}$ *and* $\vec{b}$ *is*

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}|\,|\vec{b}|}$$

$$\sin\theta = \frac{\sqrt{138}}{\sqrt{11}\,\sqrt{14}}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{\sqrt{138}}{\sqrt{11}\sqrt{14}}\right)$$

**19.(b). Find the magnitude of the torque about the point ( 4, 3, – 1) of the force represented by $6\vec{i} + \vec{j} - \vec{k}$ acting through the point ( 0 , 1 , – 1)**

**Answer** Formula: Moment of the force = $\vec{r} \times \vec{F}$

Given Acting point: = (0 ,1 , – 1)

About point: = (4, 3,– 1)

| $\vec{F}$ **(force )** | $\vec{F} = 6\vec{i} + \vec{j} - \vec{k}$ |
|---|---|
| $\vec{r}$ = *Acting point – About point* <br> $= (0, 1, -1) - (4, 3, -1)$ <br> $= (0-4,\ 1-3,\ -1+1)$ <br> $= (-4, -2,\ 0)$ | |

$$\vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & -2 & 0 \\ 6 & 1 & -1 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix} - \vec{j}\begin{vmatrix} -4 & 0 \\ 6 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} -4 & -2 \\ 6 & 1 \end{vmatrix}$$

$$= \vec{i}(2-0) - \vec{j}(4-0) + \vec{k}(-4+12)$$

$$= \vec{i}(2) - \vec{j}(4) + \vec{k}(8)$$

$$\vec{r} \times \vec{F} = 2\vec{i} - 4\vec{j} + 8\vec{k}$$

**Magnitude of the moment** = $|\vec{r} \times \vec{F}|$

$$= \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{(2)^2 + (-4)^2 + (8)^2}$$

$$= \sqrt{4+16+64}$$

Magnitude of moment $= \sqrt{84}$ *units*

**19.(c). If $\vec{a} = 2\vec{i} + 3\vec{j} + \vec{k}$, $\vec{b} = \vec{i} - 2\vec{j} + 3\vec{k}$ and**

**$\vec{c} = 3\vec{i} + 2\vec{j} - 5\vec{k}$**

**verify that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \bullet \vec{c})\vec{b} - (\vec{a} \bullet \vec{b})\vec{c}$**

***Answer*** LHS $= \vec{a} \times (\vec{b} \times \vec{c})$

Given $\vec{a} = 2\vec{i} + 3\vec{j} + \vec{k}$, $\vec{b} = \vec{i} - 2\vec{j} + 3\vec{k}$, $\vec{c} = 3\vec{i} + 2\vec{j} - 5\vec{k}$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 3 \\ 3 & 2 & -5 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} -2 & 3 \\ 2 & -5 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 3 \\ 3 & -5 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & -2 \\ 3 & 2 \end{vmatrix}$$

$$= \vec{i}(10 - 6) - \vec{j}(-5 - 9) + \vec{k}(2 + 6)$$

$$= \vec{i}(4) - \vec{j}(-14) + \vec{k}(8)$$

$$\vec{b} \times \vec{c} = 4\vec{i} + 14\vec{j} + 8\vec{k}$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 1 \\ 4 & 14 & 8 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} 3 & 1 \\ 14 & 8 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 1 \\ 4 & 8 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 3 \\ 4 & 14 \end{vmatrix}$$

$$= \vec{i}(24 - 14) - \vec{j}(16 - 4) + \vec{k}(28 - 12)$$

$$= \vec{i}(10) - \vec{j}(12) + \vec{k}(16)$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = \mathbf{10\vec{i} - 12\vec{j} + 16\vec{k}}$$

**RHS** = $(\vec{a} \bullet \vec{c})\vec{b} - (\vec{a} \bullet \vec{b})\vec{c}$

$(\vec{a} \bullet \vec{c}) = (2\vec{i} + 3\vec{j} + \vec{k}) \bullet (3\vec{i} + 2\vec{j} - 5\vec{k})$

$= (2)(3) + (3)(2) + (1)(-5)$

$= 6 + 6 - 5$

$= \mathbf{7}$

$(\vec{a} \bullet \vec{b}) = (2\vec{i} + 3\vec{j} + \vec{k}) \bullet (\vec{i} - 2\vec{j} + 3\vec{k})$

$= (2)(1) + (3)(-2) + (1)(3)$

$= 2 - 6 + 3$

$= -1$

$(\vec{a} \bullet \vec{c})\vec{b} - (\vec{a} \bullet \vec{b})\vec{c} = 7(\vec{i} - 2\vec{j} + 3\vec{k}) - (-1)(3\vec{i} + 2\vec{j} - 5\vec{k})$

$= 7(\vec{i} - 2\vec{j} + 3\vec{k}) + 1(3\vec{i} + 2\vec{j} - 5\vec{k})$

$= 7\vec{i} - 14\vec{j} + 21\vec{k} + 3\vec{i} + 2\vec{j} - 5\vec{k}$

$= \mathbf{10\vec{i} - 12\vec{j} + 16\vec{k}}$

$\therefore$ L.HS = R.H.S



**20.(a).Evaluate (i)** $\int \frac{\sin^2 x}{1+\cos x}dx$ **(ii)** $\int (\tan x + \cot x)^2 dx$

***Answer***

**(i)** $\int \frac{\sin^2 x}{1+\cos x}dx = \int \frac{1-\cos^2 x}{1+\cos x}dx$

$$= \int \frac{(1+\cos x)(1-\cos x)}{1+\cos x}dx$$

$$= \int (1-\cos x)dx$$

$$= x - \sin x + c$$

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***Answer***

**(ii) $\int (tanx + cotx)^2 dx$**

$$\int (tanx + cotx)^2 dx = \int (tan^2x + 2tanx\, cotx + cot^2x)dx$$

$$= \int \left( (sec^2x - 1) + 2tanx \frac{1}{tanx} + (cosec^2x - 1) \right) dx$$

$$= \int (sec^2x - 1 + 2 + cosec^2x - 1)dx$$

$$= \int (sec^2x + cosec^2x)dx$$

$$= tanx - cotx + c$$

| |
|---|
| **20.(b).Evaluate (i) $\int \frac{(x+1)dx}{x^2+2x-1}$ (ii) $\int \frac{\sec^2 x}{5+tanx} dx$** |

| **Answer** | **(i)** $\int \frac{(x+1)dx}{x^2+2x-1}$ ----- (1) |
|---|---|
| Put $t = x^2 + 2x - 1$<br>$\frac{dt}{dx} = 2x + 2$<br>$\frac{dt}{2} = (x + 1)\, dx$ | $(1) \Rightarrow \int \left(\frac{1}{t}\right) \frac{dt}{2} = \frac{1}{2} \int \frac{1}{t}\, dt$<br>$= \frac{1}{2}\log(t) + c$<br>$= \frac{1}{2}\log(x^2 + 2x - 1) + c$ |

| **Answer** | **(ii)** $\int \frac{\sec^2 x}{5+tanx}\, dx$ ----- (1) |
|---|---|
| Put $t = 5 + tanx$<br>$\frac{dt}{dx} = \sec^2 x$<br>$dt = \sec^2 x\, dx$ | $(1) \Rightarrow \int \frac{1}{t}\, dt$<br>$= \log(t) + c$<br>$= \log(5 + tanx) + c$ |

**20.(c).Evaluate (i) $\int \frac{dx}{25-9x^2}$ (ii) $\int \frac{dx}{(2x+3)^2+9}$**

| *Answer* | **(i)** $\int \frac{dx}{25-9x^2} = \int \frac{dx}{(5)^2-(3x)^2}$ ----- (1) |
|---|---|
| Put $t = 3x$ <br> $\frac{dt}{dx} = 3$ <br> $\frac{dt}{3} = dx$ | $(1) \Rightarrow \int \frac{\frac{dt}{3}}{(5)^2-(t)^2} = \frac{1}{3}\int \frac{dt}{(5)^2-(t)^2}$ <br> $= \frac{1}{3}\left\{\frac{1}{2(5)} log\left(\frac{5+t}{5-t}\right)\right\} + c$ <br> $= \frac{1}{30} log\left(\frac{5+3x}{5-3x}\right) + c$ |

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*Answer*

| | |
|---|---|
| | **(ii)** $\int \frac{dx}{(2x+3)^2+9} = \int \frac{dx}{(2x+3)^2+(3)^2}$ --- (1) |
| Put $t = 2x+3$ <br> $\frac{dt}{dx} = 2$ <br> $\frac{dt}{2} = dx$ | $(1) \Rightarrow \int \frac{\frac{dt}{2}}{(t)^2+(3)^2} = \frac{1}{2}\int \frac{dt}{(t)^2+(3)^2}$ <br> $= \frac{1}{2}\left\{\frac{1}{3} tan^{-1}\left(\frac{t}{3}\right)\right\} + c$ <br> $= \frac{1}{6} tan^{-1}\left(\frac{2x+3}{3}\right) + c$ |

**21.(a). Evaluate (i) $\int x^2 \log x \, dx$ (ii) $\int x \cos 2x \, dx$**

**(i) $\int x^2 \log x \, dx$**

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*Answer* **(ii) $\int x \cos 2x \, dx$**

Let $u = x$

$\frac{du}{dx} = 1$

$du = dx$

$\int dv = \int \cos 2x \, dx$

$v = \frac{\sin 2x}{2}$

Formula: $\int \boldsymbol{u\,dv} = \boldsymbol{uv} - \int \boldsymbol{v\,du}$

$$\int x \cos 2x \, dx = \frac{x \sin 2x}{2} - \int \frac{\sin 2x}{2} \, dx$$

$$= \frac{x \sin 2x}{2} - \frac{1}{2} \int \sin 2x \, dx$$

$$= \frac{x \sin 2x}{2} - \frac{1}{2}\left(\frac{-\cos 2x}{2}\right) + c$$

$$= \frac{x \sin 2x}{2} + \frac{\cos 2x}{4} + c$$

**21.(b). Evaluate: $(i)$: $\int x^2 e^{-3x} \, dx$ (ii) $\int x^2 \sin 6x \, dx$**

***Answer*** $(i)$ $\int x^2 e^{-3x} \, dx$

$\int dv = \int e^{-3x} dx$

Let $u = x^2$

$u' = 2x$

$u'' = 2$

$v = \frac{e^{-3x}}{-3}$

$v_1 = \frac{e^{-3x}}{9}$

$v_2 = \frac{e^{-3x}}{-27}$

Formula: $\int \boldsymbol{u\,dv} = \boldsymbol{uv} - \boldsymbol{u'\,v_1} + \boldsymbol{u''v_2} - \cdots$

$$\int x^2 e^{-3x} \, dx = x^2 \left(\frac{e^{-3x}}{-3}\right) - 2x\left(\frac{e^{-3x}}{9}\right) + 2\left(\frac{e^{-3x}}{-27}\right) + c$$

$$= -\frac{x^2 e^{-3x}}{3} - \frac{2x e^{-3x}}{9} - \frac{2e^{-3x}}{27} + c$$

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**Answer** **(ii)** $\int x^2 \sin 6x \, dx$

Let $u = x^2$

$u' = 2x$

$u'' = 2$

$\int dv = \int \sin 6x \, dx$

$v = \frac{-\cos 6x}{6}$

$v_1 = \frac{-\sin 6x}{36}$

$v_2 = \frac{\cos 6x}{216}$

Formula: $\int \boldsymbol{u\, dv} = \boldsymbol{uv - u' v_1 + u'' v_2 - \cdots}$

$$\int x^2 \sin 6x \, dx = x^2\left(\frac{-\cos 6x}{6}\right) - 2x\left(\frac{-\sin 6x}{36}\right) + \frac{2\cos 6x}{216} + c$$

$$= \frac{-x^2 \cos 6x}{6} + \frac{2x \sin 6x}{36} + \frac{2\cos 6x}{216} + c$$

**21.(c). Evaluate:** $\int_0^{\frac{\pi}{2}} \log(\tan x) dx$

**Answer**

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \log(\tan x) dx \quad \text{- - - - - (1)}$$

$Put\ x = \frac{\pi}{2} - x,$ in (1) we get

$$I = \int_0^{\frac{\pi}{2}} \log\left[\tan\left(\frac{\pi}{2} - x\right)\right] dx$$

$$I = \int_0^{\frac{\pi}{2}} \log(\cot x)\, dx \quad \text{- - - - - (2)}$$

Adding (1) and (2) ,we get

$$I + I = \int_0^{\frac{\pi}{2}} \log(\tan x)\, dx + \int_0^{\frac{\pi}{2}} \log(\cot x)\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log(\tan x \cot x)\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log\left(\tan x \frac{1}{\tan x}\right) dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log 1 \, dx$$

$$2I = 0$$

$$I = 0$$

$$\int_0^{\frac{\pi}{2}} \log(\tan x)\, dx = 0$$

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